Abstract—Continuous-time digital signal processors not only offer significant energy savings in important applications such as implantable biomedical devices, but can implement asynchronous procedures. In this paper, we propose an asynchronous signal decomposition for continuous-time signals based on scale rather than frequency. Because the implementation of the proposed procedure does not use a clock it is not affected by aliasing, and moreover no quantization is involved. Such procedure is specially applicable to biomedical signals delivering information in bursts rather than continuously. The decomposer consists of cascaded modules that expand the signal onto different resolution scales and each is composed of an asynchronous sigma delta modulator (ASDM) followed by a local averager and a low-pass filter. The ASDM is a non-linear feedback system used to represent the amplitude of a continuous-time signal by a binary signal whose zero-crossings are used to reconstruct the original signal. One of the parameters of the ASDM is used as a scaling parameter, permitting us to represent the signal by its local means—at different scales—and computed from the zero-crossing times of the output of the ASDM. We develop a compact signal representation that is described by a small number of scale parameters and contains information useful in the continuous-time processing and transmission of the data. The performance of the proposed procedure is illustrated using different types of signals. As a practical application, we consider the non-linear denoising of swallowing signals. Potentially our procedure will find application in asynchronous signal acquisition, continuous-time digital signal processing and transmission in low-power biomedical applications.

Energy efficiency is significant in biomedical and health-monitoring applications [3], where the replacement of batteries cannot be easily done. Asynchronous, i.e., without clocks, techniques alleviate the power consumption. Instead of using analog to digital converters to sample and quantize the samples of a signal, asynchronous systems either acquire samples only when the signal reaches certain values, as in level-crossing sampling [4], or convert the amplitude of the signal into a binary signal with zero-crossing values proportional to the signal amplitude, as in time-encoding using asynchronous sigma delta modulators (ASDM) [3], [9]. These encoders are typically implemented as low power, asynchronous analog circuits.

Using some of the properties of the time-encoding, it is possible to show that two consecutive pulses in the binary output of an ASDM can be used to determine the local average of the signal. A parameter related to the maximum frequency of the input of the ASDM can be used to regulate the scale at which these local averages are computed. By cascading several modules, each consisting of an ASDM followed by an averager and a LP filter, we obtain a decomposition of the signal into multi-level components. Moreover, when zero-mean noise is present in the signal our procedure is capable of denoising the signal without the smoothing effects caused by linear filtering. The performance is very similar to that of a non-linear filtering method such as median filtering. We develop a compact signal representation that is described by a small number of scale parameters and contains information useful in the continuous-time processing and transmission of the data.

As an application of our decomposition we consider the denoising of signals obtained from swallowing. Swallowing is a complex process of transporting food or liquid from the mouth to the stomach [11]. Patients suffering from dysphagia (swallowing difficulty), usually deviate from the well-defined pattern of healthy swallowing and are likely to aspirate. Aspiration (the entry of material into the airway below the true vocal folds) can have severe consequences including aspiration pneumonia and death [11], [12]. Current dysphagia management relies heavily on the videofluoroscopic swallowing study [11], which is not suitable for day-to-day monitoring. Cervical auscultation is a promising non-invasive tool for the assessment of swallowing disorders and enables daily monitoring of swallowing function. Cervical auscultat-
tion involves the examination of swallowing signals acquired via a stethoscope or other acoustic and/or vibration sensors during deglutition [13]. One such approach is swallowing accelerometry [14], which refers to an approach employing an accelerometer as a sensor during cervical auscultation. Swallowing accelerometry has been used to detect aspiration in several studies, which have described a shared pattern among healthy swallow signals, and verified that this pattern is either absent or delayed in dysphagic swallow signals (e.g., [15], [16], [17]). Our procedure provides denoising of the bursty signals obtained from swallowing. Linear filtering applied to healthy swallow signals, and verified that this pattern is either several studies, which have described a shared pattern among healthy swallow signals, and verified that this pattern is either absent or delayed in dysphagic swallow signals (e.g., [15], [16], [17]). Our procedure provides denoising of the bursty signals obtained from swallowing. Linear filtering applied to these high frequency signals and affected by noise would smooth out both signal and noise blurring the information as to the beginning and end of the signal.

II. TIME-ENCODING OF CONTINUOUS-TIME SIGNALS

To decrease the power consumption as well as possible data corruption, asynchronous samplers try not to use clocks simplifying their electronic design. Moreover, the sampling and quantization strategy is changed by obtaining samples only when data is present. Since no frequency considerations are made such samplers do not suffer from aliasing, and by achieving desired quantization levels they do not cause quantization error. In this section we present the necessary connection of our procedure with time-encoding using ASDMs.

A. Asynchronous Sigma Delta Modulator (ASDM)

The asynchronous sigma delta modulator (ASDM), Fig. 1, is a nonlinear feedback system consisting of an integrator and a non-inverting Schmitt trigger [3]. The ASDM maps the amplitude information of the input \( x(t) \) into a time sequence \( t_k \) which are the zero crossings of its output.

Lazar [6] has proposed a time-encoding algorithm for continuous–time signals using ASDMs. For an input \( x(t) \), such that \( |x(t)| < c \) the desired output of the ASDM is a binary signal \( z(t) \) with values of \(+b\) or \(-b\), and zero-crossing times related to the amplitude of \( x(t) \). The bias \( \pm b \) is chosen bigger than the bound of \( x(t) \) to obtain increasing/decreasing function of time when integrated. When the output of the integrator reaches a predefined values \( \pm \delta \) the output \( z(t) \) is triggered to the opposite state. How fast the triggering occurs is related to the value of \( \kappa \), which connects with the maximum frequency of the input signal. The information carried by the amplitude of \( x(t) \) is carried out by the zero-crossing times of the binary output signal \( z(t) \). The zero-crossing times as well as the design parameters \( \kappa, \delta, b \) (strictly positive real numbers) depend on the nature of the signal.

The output of the integrator in \([t_k, t_{k+1}]\) is given by

\[
y(t) = y(t_k) + \frac{1}{\kappa} \int_{t_k}^{t_{k+1}} [x(\tau) - z(\tau)]d\tau
\]

(1)

Assuming that the initial state for \([y(t_k), z(t_k)]\) is \([-\delta, -b]\), at some time \( t_{k+1} > t_k \) the output of the integrator \( y(t) \) reaches the triggering mark \( \delta \) so that according to (1):

\[
\delta = -\delta + \frac{1}{\kappa} \int_{t_k}^{t_{k+1}} [x(\tau) + b]d\tau
\]

(2)

Similarly starting with \([y(t_{k+1}), z(t_{k+1})]\) at state \([\delta, b]\), we have at time \( t_{k+2} \):

\[
-\delta = \delta + \frac{1}{\kappa} \int_{t_{k+1}}^{t_{k+2}} [x(\tau) - b]d\tau
\]

(3)

Thus, for an increasing sequence \( \{t_k\}, k \in Z \), we have from above

\[
\int_{t_k}^{t_{k+1}} x(\tau)d\tau = (-1)^k [-b(t_{k+1} - t_k) + 2\kappa\delta]
\]

(4)

According to (4), the input \( x(t) \) is connected to the zero-crossings \( \{t_k\} \) and the parameters of the ASDM. As shown in [8] the input can be recovered from the terms in the right-hand side of equation (4) by approximating the integral by the trapezoidal rule.

![Fig. 2. The parameters \( \alpha_1 \) and \( \beta_1 \) in \( z(t) \) waveform.](image-url)

A parameter critical in our decomposition is \( \kappa \). To obtain an upper bound for it, we consider that \( x(t) \) is bounded by \( c \) and that the bias, \( b \), is bigger than \( c \) then

\[
-c(t_{k+1} - t_k) \leq \int_{t_k}^{t_{k+1}} x(\tau)d\tau \leq c(t_{k+1} - t_k)
\]

Replacing (4), and solving for \( t_{k+1} - t_k \) in the above equation we obtain

\[
\frac{2\kappa\delta}{b + c} \leq t_{k+1} - t_k \leq \frac{2\kappa\delta}{b - c}
\]

An upper bound for the scaling parameter \( \kappa \) is then

\[
\kappa \leq \frac{(b + c)(t_{k+1} - t_k)}{2\delta}
\]

(5)

In the case of non-uniform sampling, a sufficient condition for reconstruction of band-limited signals is that the maximum
of \( \{t_{k+1} - t_k\} \) should be less than the sampling period \( T_s \). Thus the relationship between the bandwidth of the signal and the parameter \( \kappa \) is obtained from (5) as
\[
\kappa \leq \frac{(b + c)T_s}{2\delta} \leq \frac{b + c}{4\delta f_{\text{max}}}
\]  
(6)
where \( f_{\text{max}} \) is the maximum frequency present in \( x(t) \).

B. Calculation of Local Averages

Suppose now that we add equations (2) and (3), and let \( b = 1 \), then
\[
\int_{t_k}^{t_{k+2}} x(\tau)d\tau = \left[ (t_{k+2} - t_k) - (t_k - t_1) \right] \frac{\beta_k}{\alpha_k}
\]
If we then let \( T_k = \beta_k + \alpha_k \), then the local average
\[
\bar{x}_k = \frac{1}{T_k} \int_{t_k}^{t_{k+2}} x(\tau)d\tau = \frac{\beta_k - \alpha_k}{\beta_k + \alpha_k}
\]  
(7)
As shown in Fig. 2, \( \bar{x}_k \), the local average in \([t_k, t_{k+2}]\) corresponds to the difference of the areas under two consecutive pulses in \( z(t) \) divided by the length of the two pulses.

This result is very important in terms of power consumption efficiency of the processing system. Clearly if there is no change in the signal the output of the ASDM becomes periodic and if we detect this, there is no need to send a signal as the current input carries no new information.

In terms of our decomposition algorithm, we can easily see that a change in \( \kappa \) leads to changes in the zero-crossing times, and hence in \( \alpha_k, \beta_k \). We can consider \( T_k = \alpha_k + \beta_k \) as the width of the window that is used to estimate the local statistics, in this case the local average.

Increasing \( \kappa \) indicates that we are calculating the average of the input over a wider window. All in all, \( \kappa \) is a scale parameter that defines the width of the time interval over which we evaluate the local mean of the data.

III. ASYNCHRONOUS DECOMPOSITION

The proposed decomposition is depicted in Fig. 3. It consists of cascading \( L \) modules, each having an ASDM, an averager, a low-pass filter and an adder. The number of modules, \( L \), is determined by the scale parameters used to decompose the input signal. The ASDM maps the input signal, for a certain scale \( \kappa \), into a binary signal with sequences \( \{\alpha_k\} \) and \( \{\beta_k\} \), which the averager converts into local averages \( \{\bar{x}_k\} \). The low-pass filter is used to smooth out the multi-level signal output by the averager. By doing so, there will not be any discontinuities inserted by the adder when the multi-level signal is subtracted from the input signal of the corresponding module. Each of these modules operates similarly but at a different scale.

We start with a scale factor \( \kappa_1 \), corresponding to a wide window and obtained by considering the maximum frequency present in \( x(t) \) as indicated in equation (6). Then, we use a fraction of \( \kappa_1 \) for the other modules as the scale. The input to the modules beyond the first one can be written sequentially as follows,
\[
\begin{align*}
\bar{f}_1(t) &= x(t) - d_1(t) \\
\bar{f}_2(t) &= \bar{f}_1(t) - d_2(t) = x(t) - d_1(t) - d_2(t) \\
&\vdots \\
\bar{f}_L(t) &= x(t) - \sum_{l=1}^{L} d_l(t)
\end{align*}
\]  
(8)
where the \( d_i(t) \) are the outputs of the low-pass filters. We thus have the decomposition
\[
x(t) = \sum_{l=1}^{L} d_l(t) + \bar{f}_L(t)
\]  
(9)
where, \( \bar{f}_L(t) \) can be thought as the error of the decomposition. This decomposition does seem analogous to the wavelet decomposition, but it is valid for continuous rather than discrete signal and it uses a scale rather than frequency to implement the decomposition.

In terms of compression, the input signal \( x(t) \) can be equivalently represented by \( L \) sets of values \( \{\kappa_i, \alpha_{ik}, \beta_{ik}\} \), \( i = 1, \cdots, L \) from which we can generate the signal \( m_i(t), i = 1, \cdots, L \).

A. Simulations

Throughout our simulations we observed that the final error term, \( \bar{f}_L(t) \), goes to zero after a small number of modules \( L \). That means we can decompose the signal into continuous waveforms by using small number of scale factors. This is a promising result for data compression and transmission.
applications, beyond the advantage that neither sampling or quantization distortion is involved in the procedure.

![Effect of the scale parameter](image)

**Fig. 4.** Effect of the scale parameter \( \kappa \): (a) \( \kappa = 0.025 \), (b) \( \kappa = 0.01 \), (c) \( \kappa = 0.005 \), (d) \( \kappa = 0.00125 \).

It can be clearly seen that as the value of \( \kappa \) decreases the narrow the window is. This way the statistics of the data can be obtained in more localized intervals.

After obtaining constant amplitude continuous time binary waveforms \( z_i(t) \), \( i = 1, \ldots, L \) at the output of the ASDMs, we calculate the local averages by using the zero crossings according to equation (7). Once we obtain the local average for each interval, we form a multi-level signal where each level be obtained in more localized intervals. This way the statistics of the data can narrow the window is. This way the statistics of the data can be obtained in more localized intervals.

For each interval, we form a multi-level signal where each level according to equation (7). Once we obtain the local average for each interval, we form a multi-level signal where each level indicates the local average of the corresponding interval. Fig. 4 displays the multi-level signals at four consecutive modules for a test signal

\[
x_1(t) = \sin(2\pi t) + \sin(2\pi 10t) + \sin(2\pi 50t)
\]

\( x \). The multi-level output of the averager feeds into a low-pass filter that has a very small cut-off frequency which gets rid of the high-frequency components of the multi-level signal. The same low-pass filter is used in each module. In the actual implementation these filters would be continuous. Finally we take the difference between the input and the output signal from the filter and feed that into another identical block consisting of same elements, only with a different scale parameter. We treat the output of the each block, \( f_i(t) \), \( i = 1, \ldots, L \) as an error term since it is the difference between the original signal and decomposed parts.

To illustrate the performance of the proposed decomposition, we consider first a test signal 1 sec. long and sampled at 1 KHz (in order to simulate the continuous-time)

\[
x_1(t) = \sin(20\pi t) + \sin(10\pi t) + \sin(60\pi t)
\]

For this signal, we used the following set of \( \kappa \) values consecutively,

\[
[0.01, 0.0071, 0.0036, 0.0018, 0.0009, 0.0004, 0.0002, 0.0001]
\]

and \( L = 8 \) modules. Figure 5 displays the components \( f_i(t) \) of this signal at different modules. After the 8th module, the decomposition process is terminated since \( f_8(t) \), looks like noise as shown in Fig. 5(d). The result for the reconstruction of the same signal with added noise (SNR= 3 dB) is plotted in Fig. 6.

As a second simulation illustrating the procedure, we process a bursty signal, reconstruct it and obtain the absolute error which are displayed in Fig. 7. Figure ?? shows the reconstruction of the same signal under noise.

If we disregard the last term in the decomposition, equation(8), we observe a smoothing of the noise that is embedded in the signal. This effect can be seen from Fig. 7 and Fig. 8. The algorithm smooths out the noise by using appropriate \( \kappa \) values.

**B. Application of the decomposition to de-noising of swallowing data**

In the analysis of swallowing data is of interest to determine the starting and the end of the swallowing so difficulties in swallowing are detected. Because of the bursty nature of the swallowing signals, as well as the presence of noise and other

![Input to the (a) first, (b) second, (c) 5th and (d) 8th module](image)

**Fig. 5.** Input to the (a) first, (b) second, (c) 5th and (d) 8th module, respectively.

![Reconstruction of test signal](image)

**Fig. 6.** Reconstruction of test signal \( x_1(t) \) (dark signal) with additive noise SNR=3 dB.
1) Swallowing Data: The recordings considered here were collected over a three month period from a public science center in Toronto, Ontario, Canada. To monitor vibrations associated with swallowing, a dual-axis accelerometer (ADXL322, Analog Devices) was attached to the participant’s neck (anterior to the cricoid cartilage) using double sided-tape. The axes of the accelerometer were aligned to the anterior-posterior and superior-inferior directions. The research ethics boards of two hospitals in Toronto, Ontario, Canada (Holland Bloorview Kids Rehabilitation Hospital and Toronto Rehabilitation Institute) approved the study protocol. All participants provided written consent and had no documented swallowing disorders. Additionally, all participants passed an oral mechanism exam prior to participation.

Data were sampled at 10 kHz and collected using a custom LabVIEW program running on a laptop computer. Hardware-based band-pass filters were also used with a pass band of 0.1 – 3000 Hz. Each participant performed three types of swallows involving saliva and water swallows. The entire session lasted for about 15 minutes per participant.

2) Results: The sample results shown in Figs. 9 and 10 were obtained using $\kappa = [0.01, 0.005, 0.0025, 0.0013, 0.0006]$ and $L = 5$ modules. These results clearly depict that the artifacts, denoising these signals is of great interest. Linear filtering does not work well given the high-frequency nature of the swallowing signals. The proposed decomposition works like a non-linear filter given its localized processing.
proposed scheme can provide a denoised version of swallowing accelerometry signals. Specifically, the location and the duration of swallows is clearer from the synthesized signals. Such denoised signals can be then used to segment these recordings more accurately. This is particularly obvious in Fig. 10 where the swallows are not easily observable in the original signal. Specifically, the presence of five swallows is obvious from the synthesized signal.

IV. CONCLUSION

A scale-based continuous-time decomposition technique is presented and its performance illustrated using different types of signals. It is shown that fine decomposition can be obtained as long as scale factor keeps under an upper bound defined by the bandwidth of the signal. We have also shown how input signal decomposition can lead to smoothing of the noise present in the signal. By operating in continuous time, the system avoids aliasing and quantization error. The simulations illustrate the application of our decomposition for signal compression, and denoising. As an application, we consider the denoising of bursty signals obtained from swallowing.

REFERENCES