A scaling exponent-based detector of chaos in oscillatory circuits

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Abstract

In this paper, a scaling exponent based approach is proposed to determine the state of chaotic circuits, and the scaling exponent is calculated using detrended fluctuation analysis (DFA). The corresponding detector is designed using the fact that the scaling exponent changes for various states of chaotic circuits. Simulation examples in this paper are performed for the Chua's circuit and other chaotic systems and compared with the state-of-the-art in the field. The proposed detector outperforms existing techniques in ability to distinguish the chaotic and periodic states in the circuits for relatively high noise.

Keywords: Chaos detection, Scaling exponent, Detrended fluctuation analysis, Peak intervals time series.

1. Introduction

Chaotic nonlinear oscillators belong to a group of systems that can exhibit chaotic behavior [1]-[4]. Estimating a current state of chaotic systems based on a available time series is an active research area.

In the past decades, there are classical works reporting how to detect chaos from times series, such as Lyapunov methods [5], [6], the Grassberger-Procaccia algorithm (GPA) for estimation of the correlation dimension [7],

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the Kolmogorov entropy [8], etc. These methods are based on quantification of the nearest neighbors in the phase space. Therefore, these techniques are computationally expensive.

The scope of this paper is a subset of the more general topic of detecting dynamic changes and weak transitions in signal dynamics. A number of interesting methods have been proposed to detect dynamic changes. Among others, methods using recurrence dynamic systems analyzing a number of visits to small regions in the phase space such as: recurrence plots [9], recurrence quantification analysis [10], recurrence time statistics based approaches [11], [12]. Again, the majority of these techniques analyzes the nearest neighbor in the phase space. The permutation entropy was proposed at beginning of the last decade as an interesting concept for measuring complexity of time series [13]. This concept is simple and leads to efficient evaluation [14].

In other direction, techniques based on the time-frequency (TF) signal analysis have been proposed in [15]. Namely, the period-doubling, torus breakdown and intermittency routes to chaos have well-defined spectral representations and it is possible to distinguish between chaotic and periodic system states in this domain. For example, a detector of chaotic states in nonlinear oscillatory circuits based on the short-time Fourier transform (STFT) has been proposed in [15]. This detector has ability to distinguish chaotic and noise-contaminated samples when moderate level of noise is present. The accuracy of such a detector with respect to the window width used for the TF representation is analyzed in [16], resulting in the multiple STFT-based approach with a higher accuracy than the single window counterpart. However, the STFT-based detectors do not always properly differentiate chaos and noise for low signal-to-noise ratio (SNR) values. This fact reduces the application of the STFT-based detectors on chaotic signals for high SNR values.

The next approach is based on the concept of scale. The concept of scale is suitable for the analysis of signals from complex dynamic systems [17]-[20]. Specifically, the detrended fluctuation analysis (DFA) [21] is used in this paper. The DFA is an efficient scaling method used for detecting correlations in noisy, nonstationary time series [21]-[23]. It has been used widely [24], [25]. Chaotic signals seem random and unpredictable and have statistical properties similar to random processes [26]. Therefore, the DFA is suitable for the analysis of such signals [27], [28] in order to detect the state of chaotic oscillators. Furthermore, the DFA has been also used in the analysis of complex, medical signals such as heartbeat time series [29] and stride interval dynamics [30]. The idea of analyzing intervals between extremum points (or interbeat) also found its applications in other physiological signals such as those coming from brain [31] and muscle activities [32]. These results motivated us to consider complex dynamics of sequence of intervals between extremum points coming from signals associated with chaotic oscillatory circuits using the DFA. Therefore, we propose an algorithm based on the analysis of scaling exponents, calculated by DFA, associated with time series consisting of intervals between extrema in the original signal. Our analysis shows that the scaling exponent of extrema points intervals sequences significantly differs for periodic and chaotic signals. A state of the system is estimated without a priori knowledge of the structure and parameters of the oscillator. Simulation examples are done for the well-examined Chua's oscillator and other chaotic systems. The proposed algorithm has reasonable complexity and it requires short interval for evaluation measure, making it suitable for implementation in a real time. Also, the proposed algorithm is robust to significantly higher noise levels than the existing techniques. Considering shortcomings of existing techniques for the detection of chaos (high complexity of computation, inapplicability to a wide class of systems, the unreliability for short noisy sequences), the problem of detection is considered open and any new contribution to this challenging problem is important. According to the presented results, the proposed DFA-based approach is a valuable tool for the considered application.

The paper is organized as follows. In Section 2, procedure of estimation of scaling exponents is described in details and the detector of chaos in oscillatory circuits based on the DFA is proposed. The DFA is applied to typical periodic and chaotic signals from Chua's oscillator and rationale for selecting threshold and algorithm setup is described in Section 3. Simulation results and detailed analysis of noise influence on the accuracy of the detector as well as comparison with permutation entropy are given in Section 4. Concluding comments are given in Section 5.

2. Detecting chaos using scaling exponents

In this section, we propose an algorithm for detection of chaotic episodes based on the scaling exponents. It should be mentioned that the DFA is not directly applicable to the chaotic signals due to the so-called crossover effects. Therefore, the scaling exponents are obtained for the time series representing intervals between local extremum points calculated for signals from consider circuits. This is explained in details in the next section.

The steps of the proposed algorithm are as follows:

1. Obtain local extremum points:

$$S = \{n | [x(n) > x(n-1) \wedge x(n) > x(n+1)] \vee$$
$$[x(n) < x(n-1) \wedge x(n) < x(n+1)]\} . \quad (1)$$

2. Create ordered sequence of elements from S:

$$P(k) \in S, \ P(k) < P(k+1) . \quad (2)$$

Corresponding time series of intervals between extremum points is:

$$\tilde{P}(k) = P(k+1) - P(k) . \quad (3)$$

3. This sequence is divided in overlapping blocks centered around considered instant:

$$y(i) = \tilde{P}\left(k + i - \frac{N}{2}\right), \ i \in [1, N] , \quad (4)$$

where N is the length of the window. Index k is related to the time instant corresponding to local extrema point.

4. Calculate the scaling exponent using DFA.
To estimate the scaling exponent of a time series $y(i)$, $i = 1, ..., N$, we take the following steps:

(a) The time series mean is computed as $\bar{y} = \frac{1}{N}\sum\limits_{j=1}^{N} y(j)$. An integrated time series $x(i)$, $i = 1, ..., N$ is obtained as:

$$x(i) = \sum_{j=1}^{N} y(j) - \bar{y}, i = 1, ..., N. \quad (5)$$

(b) The integrated series $x(i)$ is then divided into non-overlapping segments (windows) of equal size n. A polynomial function of degree l, marked by $x_m(i; n)$, is used to interpolate the sequence in each segment. The interpolating curve $x_m(i; n)$ represents the local trend in each segment. Linear interpolation, $l = 1$, is commonly used.

(c) The fluctuation sequence, or the difference between the integrated sequence and the local polynomial trend is calculated:

$$z_l(i; n) = x(i) - x_l(i; n) , i = 1, ..., N. \quad (6)$$

(d) The root mean square (RMS) values of fluctuations is calculated for each segment. That is fluctuation functions labeled as $F_l(n)$:

$$F_l(n) = \sqrt{\frac{1}{N}\sum_{j=1}^{N} z_l(j;n)^2}\,. \tag{7}$$

(e) This procedure is repeated for a wide range of segment length n ($n_{\min} = 5$ and $n_{\max} = \frac{N}{4}$ [21]). Note that the wider segments cause smaller number of sliding windows following with the large the RMS fluctuation around the regression line.

(f) Finally, if we assume that the signal satisfies a scaling law, it is noted that fluctuation function satisfies a power law in relation to the segment length:

$$F_l(n) \propto n^{\alpha}, \tag{8}$$

where the scaling exponent α can be computed as the slope of the plot $\log(F_l(n))$ versus $\log(n)$. The state of system is estimated based on the scaling exponents α. We perform the linear interpolation of the plot $\log(F_l(n))$ versus $\log(n)$ (in our example for $n \in [5, \frac{N}{4}]$) in order to estimate α. This slope is used as a measure for chaos detection.

5. For the time instant t, set $m(t) = \alpha$.
6. Compare $m(t)$ to a threshold C:

$$\begin{aligned} m(t) \geq C, &\quad \text{oscillatory circuits in instant } t \\ &\quad \text{is in chaotic regime,} \\ m(t) < C, &\quad \text{oscillatory circuits in instant } t \\ &\quad \text{is in periodic regime.} \end{aligned} \tag{9}$$

$m(t)$ is higher for the chaotic than for the periodic regime. In other words, if $m(t)$ is above some properly selected threshold, than the considered signal is chaotic. Otherwise, if $m(t)$ is below the threshold, we can conclude that the signal represents a periodic regime. In this process, determining a proper value of the threshold is crucial. In the next section, we will describe how to choose a proper value of threshold for the Chua's circuits.

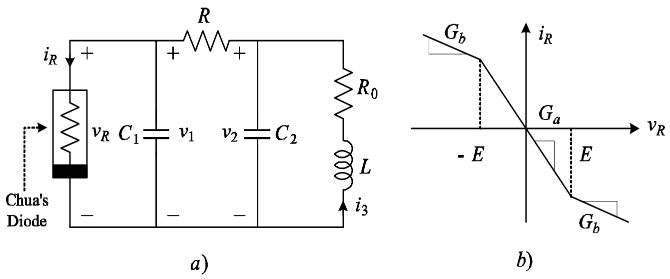

Figure 1: a) Chua's circuit; b) nonlinear v-i characteristic of Chua's diode.

3. Algorithm setup

Here, the threshold selection procedure is explained on the simple the Chua's oscillator, shown in Fig.1a. It is the simplest electronic circuit that meets all conditions for producing chaotic behavior [33]. The characteristics of a nonlinear resistor called Chua's diode is shown in Fig.1b. The circuit can be described by a set of three autonomous state equations:

$$\frac{dv_1}{dt} = \frac{1}{C_1}\left[G\left(v_2 - v_1\right) - f\left(v_1\right)\right]$$

$$\frac{dv_2}{dt} = \frac{1}{C_2}\left[G\left(v_1 - v_2\right) + i_3\right]$$

$$\frac{di_3}{dt} = \frac{1}{L}\left(-v_2 - R_0 i_3\right), \tag{10}$$

where $G = 1/R$ and $f\left(v_1\right)$ is the piece-wise linear $v - i$ characteristic of the Chua's diode, given by:

$$f\left(v_R\right) = G_b v_R + \frac{1}{2}\left(G_a - G_b\right)\left(\left|v_R + E\right| - \left|v_R - E\right|\right), \tag{11}$$

where E is the breakpoint voltage of Chua's diode.

By selecting various values of parameters, this oscillator can exhibit different types of behavior in a steady state (the equilibrium point, periodic motion, quasiperiodic motion and chaos). This circuit can also become chaotic in three different ways: period-doubling, torus breakdown and intermittency routes to chaos. That transition or route to chaos is achieved by varying one of the parameters of the circuit, while others remain constant. Table 1 shows the constant and varying parameters of Chua's circuit that

Routes to chaos	**Fixed parameters**	**Varying parameter**
Period-doubling	$L = 18$mH, $C_1 = 10$nF, $C_2 = 100$nF, $G_a = -757.576\mu$S, $G_b = -409.091\mu$S, $E = 1$V, $R_0 = 12.5\Omega$	G linearly increases from $G = 530\mu$S to $G = 565\mu$S and after that decreases toward initial value.
Torus breakdown	$L = 7.682$mH, $C_2 = 0.3606\mu$F, $G_a = 0.599$mS, $G_b = 0.77$mS $E = 1$V, $R_0 = 13.4\Omega$, $G = -0.7$mS	C_1 linearly decreases from $C_1 = 0.0297\mu$F to $C_1 = 0.008\mu$F and after that increases toward initial value.
Intermittency	$G_a = -0.756$mS, $G_b = -0.409$mS, $L = 37.56$mH, $C_2 = 215$nF, $R_0 = 30\Omega$,$E = 1$V, $G = 0.648$mS	C_1 linearly varies from $C_1 = 19.28$nF to $C_1 = 19.246$nF.

Table 1: Routes to chaos for Chua's circuit.

we used in our simulations for three mentioned routes to chaos [33], [34], [35].

The DFA is applied on the signal from the Chua's circuits for quantification of the circuit state based on obtained scaling exponents. Scaling exponents for signal from the Chua's circuit are given in Fig. 2. Solid line represents the DFA for the chaotic signal while dashed line corresponds to the periodic signal. Obviously, the fluctuation function cannot be described by the simple rule since the unique value of α cannot be determined for entire function. Namely, there is a crossover point in the fluctuation function where the scaling parameter changes value. From Fig. 2 it can be concluded that the main difference between periodic and chaotic regimes is in different exponents after the crossover point. The slope of the curve for the periodic regime is approximately zero while for the chaotic regime it has slope different from zero.

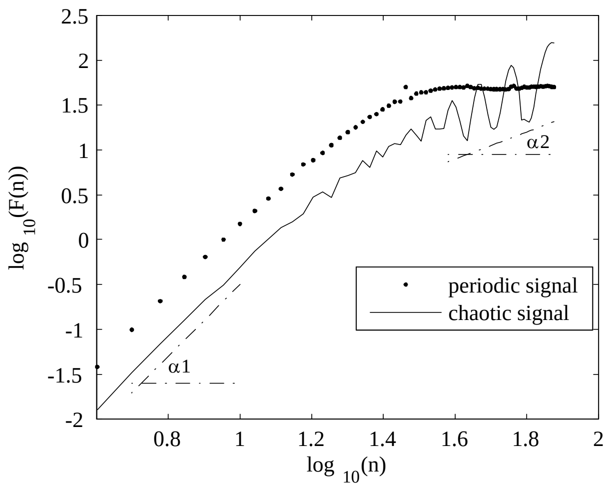

Figure 2: Result of DFA: periodic signal - dotted line, chaotic signal - solid line.

Narrow windows (below crossover point) exhibits similar behavior for chaotic and periodic regimes. Namely, periodic signal in window that is shorter than signal period has pseudorandom appearance and the window lengths below the crossover point are not useful in detection. After the crossover point the slope of the fluctuation function for periodic regime is approximately zero while for chaotic regime it is different from zero. Design of the detector in this case would be rather complicated since it requires estimation of the crossover point that is different for various routes to chaos, chaotic systems and also it depends on the sampling rate (smaller sampling rate means larger number of samples and larger value of the crossover point). Therefore, we decided to use the interval between extrema points in original signal in the DFA analysis (1)-(3). Firstly, these signals are less complex than corresponding outputs of the oscillators. For non-noisy monocomponent periodic signal this series is constant while for multicomponent it has several values with periodic appearance. For circuits in our analysis (Chua's circuit, Colpitts oscillator, Duffing oscillator, Bonhoeffer-van der Pol oscillator, Lorenz and Rossler chaotic system) this period is smaller than 5 samples. It means that n corresponding to the crossover in the DFA is less than 5 meaning that the slope of the DFA should be estimated for $n > 5$. For periodic regime this slope is equal to zero while for the chaotic region the peak interval time series produces irregular sequence giving non-zero slope for $n > 5$. Then the proposed detector will be based on the DFA analysis

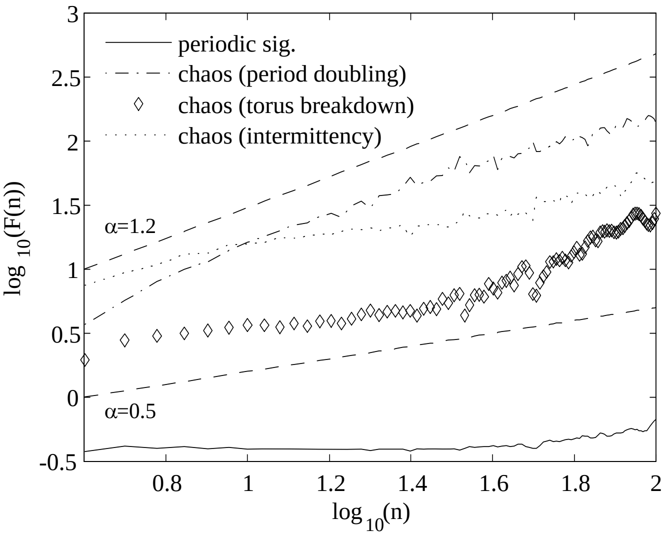

Figure 3: Result of DFA: periodic signal - solid line, chaotic signals - dashed lines.

of the peak interval time series and slope of the corresponding fluctuation function. Scaling exponents for the peak intervals time series derived from signals from the Chua's oscillators are shown in Fig. 3. The solid line represents the fluctuation function for the periodic signal. The dashed lines correspond to the fluctuation functions for the chaotic signals from three different routes to chaos: period-doubling (dashed-dotted line), torus breakdown (diamond line), intermittency (dotted line). Fig. 3 shows that $\alpha \approx 0$ for the periodic regime, and $\alpha > 0.5$ for the chaotic regime. Those results are consistent with the ones given in [27] for medium time scale which are being used in the proposed detector.

Difference between slopes in the chaotic and periodic regime is large enough to set safe threshold between these regimes. Therefore, we set the threshold to $C = 3/8 = 0.375$ in our simulations, which yields a robust performance of the detector.

The window width N cannot be too small since in that case we cannot apply the DFA analysis to the considered segment. However, wide windows are also not suitable for the considered analysis, since they could include samples from several consecutive chaotic and periodic segments. In our simulations, N was set to 100 samples but similar results are obtained with $N \in [75, 200]$.

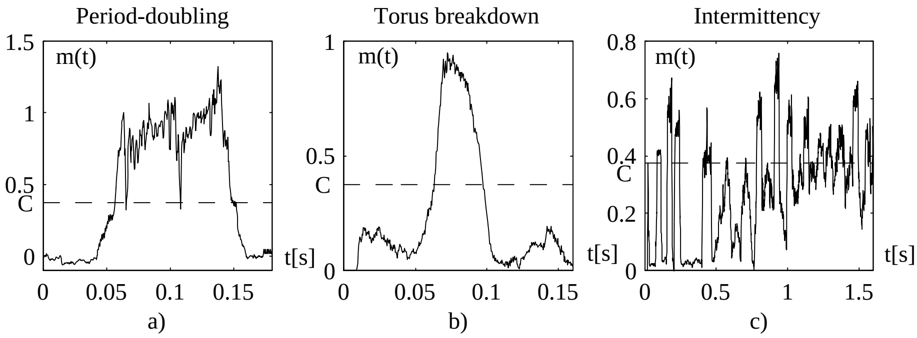

Figure 4: Detector responses for different routes to chaos while considering the Chua's circuit. The dashed line represents the detection threshold.

4. Simulation study

In this section, we examine the performance of the proposed detector for several chaotic circuits including the Chua's oscillatory circuit, the Colpitts oscillator and well-known Rossler, Lorenz, Duffing and Bonhoeffer-van der Pol chaotic systems. In addition, we consider the proposed detector for the noisy environment and compare it with several existing techniques.

4.1. Chua's oscillatory circuit

To analyze the performance of the detector for the Chua's oscillator, we consider time series of intervals between extrema points derived from $v_1(t)$. The parameters used for simulating the oscillator are shown in Table 1. Continuously varying bifurcation parameters, the chaotic circuit switches from a periodic to a chaotic state and vice versa. The length of sequence used in the DFA is 1096 samples. The DFA is performed for each instant with the analysis window of 100 samples centered around the considered instant. The DFA is calculated for sequences of length from 5 to 25 samples. Figs. 4a-c depict the measure $m(t)$ with comparison to threshold C for period-doubling, torus breakdown and intermittency routes to chaos, respectively. The chaotic regime, indicated with values above the threshold, corresponds well with the theoretical expectations and experimental results found in [15], [33]-[35]. Also, all periodic windows are properly detected. Similar results can be obtained by using other signals (e.g., $v_2(t)$ and $i_3(t)$) from the circuits.

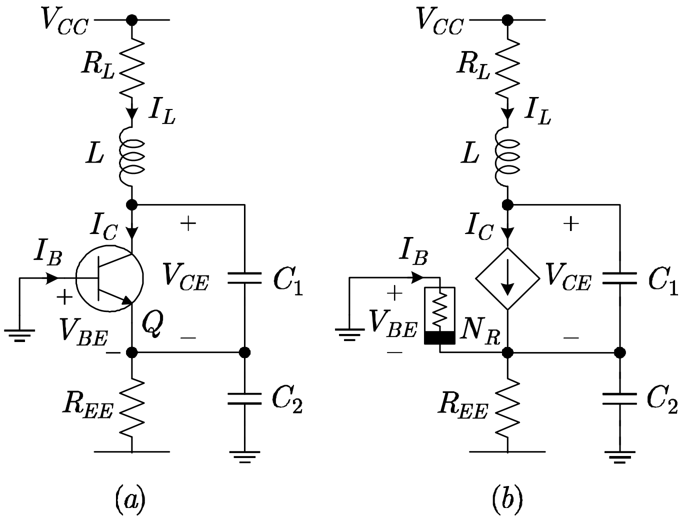

Figure 5: a) Colpitts oscillator with bipolar transistor. b) Equivalent circuit.

4.2. Others oscillators and chaotic systems

The performance of the proposed detector is also investigated for the Colpitts oscillator, given in Fig.5a, [2]. The bipolar transistor, assuming that it operates in directly active or cutoff regimes, can be modeled as a two-segment piecewise linear voltage controlled resistor (Fig.5b). Furthermore, the detector can also be used for other chaotic systems including the autonomous systems such as the Rossler and Lorenz chaotic systems and nonautonomous systems such as the Duffing oscillator and forced Bonhoeffer -van der Pol oscillator. Table 2 summarizes the equations and parameters used for the simulations of the aforementioned systems.

The results of simulations for these four systems can be seen in Fig. 6. Chaotic regions (above the threshold) and periodic regions (below the threshold) detected by the proposed detector correspond well to the theoretical expectations, [2], [15], [36]-[41].

4.3. Noise influence

The main advantage of the proposed technique is its robustness to noise, since the previously proposed techniques based on the time-frequency representations are able to distinguish the chaos from noise only for small-to-moderate levels of the additive noise. In order to analyze the influence of the additive noise, we add the Gaussian noise with a varying variance. The

Equations	**Fixed parameters**	**Varying parameter**
Colpitts oscillator $C_1 \frac{dv_{CE}}{dt} = i_L - I_C$ $C_2 \frac{dv_{BE}}{dt} = -\frac{V_{EE}+v_{BE}}{R_{EE}} - i_L - I_B$ $L \frac{di_L}{dt} = V_{CC} - v_{CE} + v_{BE} - i_L$ $I_B = \begin{cases} 0 & v_{BE} \leq V_{TH} \\ \frac{v_{BE}-V_{TH}}{R_{ON}} & v_{BE} > V_{TH} \end{cases}$ $I_C = \beta_F I_B$	$C_1 = 54$nF, $C_2 = 54$nF, $L = 98.5\mu$H, $R_{EE} = 400\Omega$, $V_{EE} = -5V$, $V_{CC} = 5V$, $\beta_F = 255$, $R_{ON} = 100\Omega$, $V_{TH} = 0.75V$.	Linearly varying R_L from 67Ω to 5Ω.
Rossler chaotic system $\frac{dx}{dt} = -y - z$ $\frac{dy}{dt} = x + ay$ $\frac{dz}{dt} = b + z(x - c)$	$a = b = 0.2$	c increases from 2 to 5.7.
Lorenz chaotic system $\frac{dx}{dt} = -\sigma x + \sigma y$ $\frac{dy}{dt} = -xz + rx - y$ $\frac{dz}{dt} = xy - bz$	$\sigma = 10$ $b = 8/3$	r increases in range $10 < r < 110$.
Duffing oscillator $\frac{dx}{dt} = y$ $\frac{dy}{dt} = x - x^3 - \delta y + \gamma \cos(\omega t)$	$\delta = 0.5$	γ decreases from $\gamma = 0.88$ to $\gamma = 0.7$.
Bonhoeffer-van der Pol oscillator $\frac{dx}{dt} = x + \frac{x^3}{3} - y + A_1 \cos(t)$ $\frac{dy}{dt} = c(x + a - by)$	$a = 0.7$ $b = 0.8$ $c = 0.1$	A_1 linearly increases from $A_1 = 0$ to $A_1 = 0.78$ and after that decreases toward initial value.

Table 2: Parameters of chaotic systems.

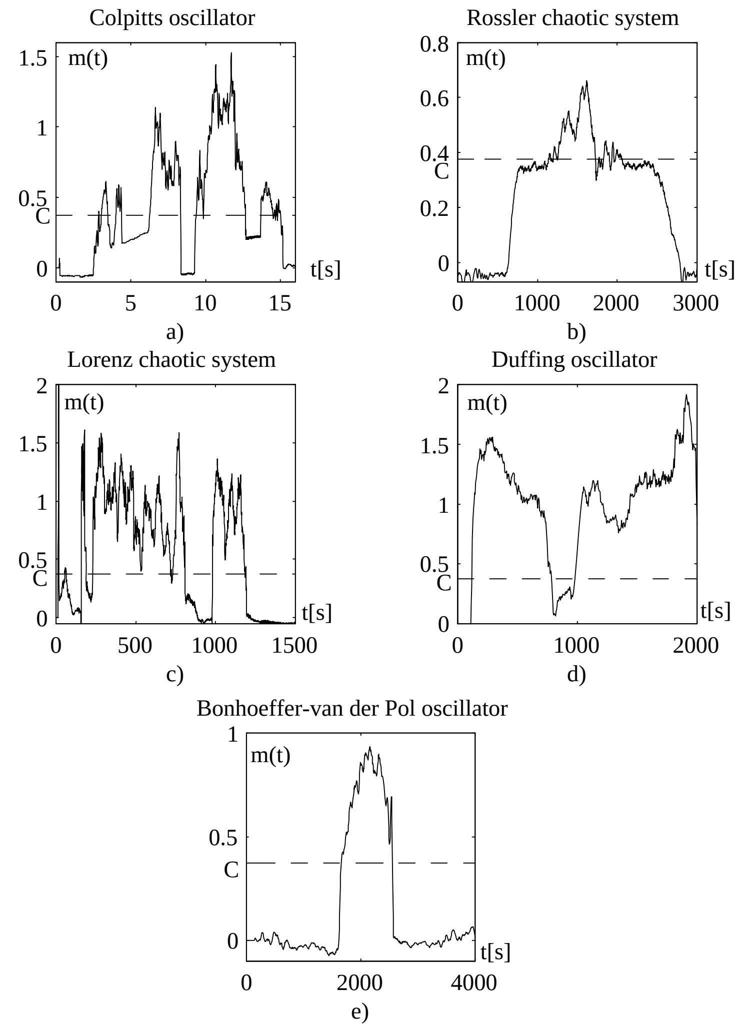

Figure 6: Detector responses for other chaotic systems. Detection threshold–dashed line.

SNR is evaluated as [42]:

$$SNR = 10\log_{10}\frac{\int_t |f(t)-\overline{f}(t)|^2 dt}{\int_t |\nu(t)|^2 dt}, \quad (12)$$

where $\overline{f}(t)$ is the signal mean value in the short interval around considered instant (in our case, it is 100 samples wide interval):

$$\overline{f}(t) = \frac{1}{T}\int_{t-T/2}^{t+T/2} f(\tau)d\tau. \quad (13)$$

while $\nu(t)$ is additive noise. Due to the presence of noise, additional local extreme values appear and the sequence consisting of intervals between extrema points significantly differ from those obtained in a noise-free case. Therefore, we use a lowpass filter (cutoff frequency is $f_c = \frac{f_{\max}}{4}$, where $f_{\max} = \frac{1}{2T}$, with sampling rate T) to remove high frequency components associated with noise.

To analyze the proposed detector, we compare its performance with the detector proposed in [15] for SNR values in the range of $[0, 20]$dB. The parameters shown in Table 1 are used for period-doubling route to chaos. The accuracy of the proposed detector is tested using the Monte Carlo simulation. Each resulting value is obtained using 100 trials. A detection error is defined as the percentage of the samples from the periodic regime missclassified as chaotic. This definition stems from the fact that some instants of the periodic regime for a noisy signal could be recognized to belong to the chaotic region, while the opposite is rarely the case. In other words, noise can push measures for periodic regime above the threshold due to similarity between chaos and noise. The proposed detector gives quite accurate results with less than 17% missclasified samples for the considered SNR range. It is slightly worse than detector from [15] for high SNR (SNR>14dB) while it is significantly better for SNR<10dB. In this way the range of the SNR values for which the proposed algorithm works accurately is significantly extended comparing to the existing techniques. This is the main purpose of the proposed technique, since distinguishing between chaos and noise influence is a major difficulty in this area. The proposed detector is less accurate than [15] only for samples close to the chaotic region. The scaling exponent is calculated based on the samples from both chaotic and periodic regimes, which reduces the accuracy of the estimator. However, this issue is beyond the scope of the current manuscript. Note that similar results are obtained for other routes to chaos and other chaotic circuits.

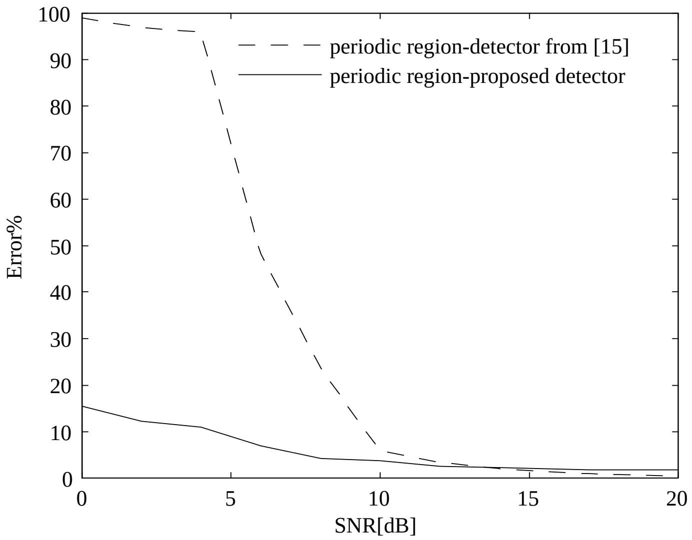

Figure 7: Percentage of detector error with proposed algorithm and with algorithm from [15].

4.4. Comparison with permutation entropy

Next, we compare the proposed technique with the permutation entropy approach (e.g., [14]) for the Lorenz chaotic system with bifurcation parameter r varying from 28 to 268. The results are shown in Fig. 8. The proposed detector is capable of differentiating between chaotic and periodic regimes as shown in Fig. 8c. However, to set up the threshold or procedure for distinguishing between the chaotic and periodic regimes for the permutation entropy based detector can be a challenging task as depicted in Fig. 8b. In addition, the proposed detector is significantly more robust to noise in comparison to the permutation entropy. Namely, there are no significant dynamic variations in the permutation entropy at $SNR = 15$dB (these variations should indicate changes in the system regime), while the proposed estimator still works accurately (see Fig. 7).

5. Conclusion

In this paper, the scaling exponent-based chaotic regime detector for signals from oscillatory circuits was proposed. The detector was developed using the scaling analysis of extrema point intervals time series. The proposed technique has been tested for three the most common routes to chaos:

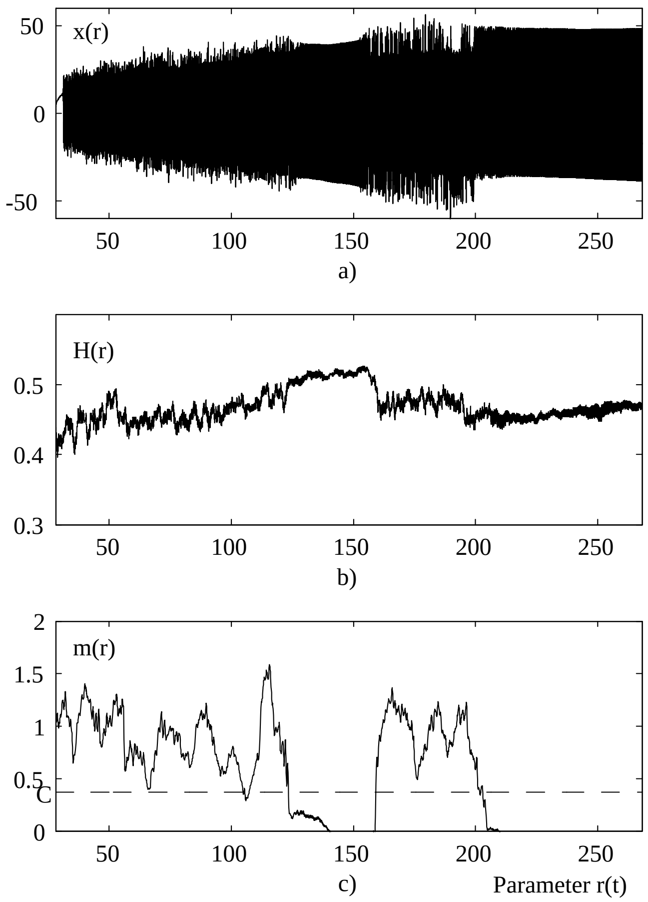

Figure 8: a) The transient Lorenz data. b) The detector response for permutation entropy $H(r)$ $(m = 4, L = 10)$. c) The response of the proposed scaling exponent-based detector. The dashed line denotes a detection threshold.

period-doubling, torus breakdown and intermittency. Also, this technique was successfully applied in different autonomous and nonautonomous chaotic systems. A numerical analysis indicated that the proposed algorithm was very accurate and robust to noise influence in comparison to other existing techniques.

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