

Analysis of the Pen Pressure and Grip Force Signal During Basic Drawing Tasks: the Timing and Speed Changes Impact Drawing Characteristics

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Abstract

Writing is a complex fine and trained motor skill, involving complex biomechanical and cognitive processes. In this paper, we propose the study of writing kinetics using three angles: the pen-tip normal force, the total grip force signal and eventually writing quality assessment. In order to collect writing kinetics data, we designed a sensor collecting these characteristics simultaneously. Ten healthy right-handed adults were recruited and were asked to perform four tasks: first, they were instructed to draw circles at a speed they considered comfortable; they then were instructed to draw circles at a speed they regarded as fast; afterwards, they repeated the comfortable task compelled to follow the rhythm of a metronome; and eventually they performed the fast task under the same timing constraints. Statistical differences between the tasks were computed, and while pen-tip normal force and total grip force signal were not impacted by the changes introduced in each task, writing quality features were affected by both the speed changes and timing constraint changes. This verifies the already-studied speed-accuracy trade-off and suggest the existence of a timing constraints-accuracy trade-off.

Keywords: writing kinetics, sensor design, feature extraction, signal features

1. Introduction

Handwriting is both a fine and well trained motor skill, and it involves several complex biomechanical processes along with cognitive and sensory abilities [15, 21, 7]. The complexity of the systems involved in the writing process, along with its trained nature, results in variations between and within subjects [21]. The biomechanical processes entailed in handwriting is more particularly the result of the coordination of the arm-finger biomechanical system, which involves more than ten mechanical degrees of freedom [1]. However, this high degrees of freedom system can be simplified to a process involving only two degrees of freedom, related to the motion of the fingers and the wrist, and writing can consequently be modeled with

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two oscillation motion generators [1]. These oscillations generators make handwriting particularly related to event timing in the human body. In our paper, drawing and handwriting will be used equivalently.

The regulation of the precise timing required by skilled motion such as handwriting involves the cerebellum [12]. The role of the cerebellum in the realization of timing constrained tasks, more particularly when the production or detection of periodic events is required, as been proven to be of great importance by numerous studies. Indeed, people with cerebellar lesions were shown to have problem completing this kind of timing related tasks [10, 26, 19, 25, 18]. A repetitive task such as drawing continuous circles [12] is consequently of great interest, because of the dual aspects of timing handling the subject has to manage: the periodicity of the circle rhythm, along with the two biomechanical motion oscillators associated with writing or drawing [1, 12]. In order to get a better understanding of how timing constraints affects the subjects' realization of this task, we introduced both free and forced periodic circle drawing at both a comfortable (also referred as slow speed in this paper) and fast speed.

The studies presented here-in above focused primarily on writing kinetics as a whole (i.e. the system's input is the arm motion and the output is the pen tip motion). However, some recent work focused on the pen-finger kinetics and showed its impact on handwriting quality [6, 24, 5, 22]. Indeed, studies revealed that static grips were associated with poorer handwriting quality, while dynamic tripod like grips were associated with a higher writing proficiency [5, 22]. Studies also showed that handwriting characteristics combined with grip-force features can be used to determine stroke characteristics [6].

The emergence of computer instrumentation to access handwriting features resulted in numerous studies, each of them focusing on different aspects of the handwriting process: the use of both normal pen pressure and grip force kinetics to study the impact of strokes on handwriting [6], the study of the effect of the circadian circle of the position and normal force of the pen tip [13], or the impact of writer cramp on both grip and handwriting quality characteristics [2].

In this paper, we collected data with the objective of increasing the understanding of the timing problematic in handwriting, exploring the impact of different writing task on the grip force kinetics and eventually build a solid baseline available for further studies. Our hypothesis is that timing and speed changes will result in writing changed, and in order to verify this hypothesis, we designed an experiment in which both pen-tip normal force and grip force were gathered during 4 distinct circle-drawing tasks: the drawing of continuous circle at a comfortable speed and at a fast speed without any timing constraints, and the repetition of these two tasks with a timing constraint induced by the use of a metronome.

The paper is organized the following way: Section 2 describes the design of the sensor and the data collection process. In Section 3, the feature extraction performed on the signals is explained. The results of our experiment are presented in Section 4, while Section 5 provides a discussion of these results.

2. Methodology

2.1. Data Collection

In this study, both writing kinetics features and electroencephalogram (EEG) signals were collected on 10 healthy right-handed adults (9 males, 1 female). Their age ranged from 20 to 29. The subject provided information about height, weight and gender, and subjects signed the consent form approved by Institutional Review Board at the University of Pittsburgh.

We collected normal pen-tip force using a CTL460 Bamboo Pen Tablet (Wacom, Japan) and a grip force sensor using a force sensitive resistor (FSR) ThruMode Matrix Array (Sensitronics, Bow, WA, USA) was designed. The design details are given herein-bellow. Data from the tablet was collected using a dedicated computer software programmed especially for this task at a sampling rate between 80 Hz and 120 Hz, depending on the computer load. Data from the pressure sensor was acquired using LabView 2010 and a NI USB-6210 acquisition card (National Instruments Corporation, Austin, TX, USA) at a sampling rate of 20 Hz. Then, data from the tablet and from the FSR were synchronized, data from the tablet was resampled to match the FSR sampling frequency and features were extracted. Along with the writing kinetics features, EEG signals were collected using 64 electrodes arranged following the 10-20 international electrode system standard (not used in this paper).

The experiment consisted of four distinct tasks: all the tasks lasted 1 minute; subjects were asked to freely draw circles at a speed they considered as comfortable; a similar task was repeated except subjects were asked to draw at a self-determined fast speed; they were then asked to draw circle at a slow speed following the rhythm of a metronome (the metronomes beat per minutes were set to be the same as in the slow task); and the fourth task consisted of a similar task than the previous one, except that the BMP of the metronome were set to match the fast drawing circle rhythm. During all the circle-drawing tasks, the position of the subjects elbow was the same.

2.2. Sensor Design

The FSR used in our experiment consists of 160 pressure sensing cells arranged in 16 rows and 10 columns. Each of the cell is accessible using a (column, row) physical address. A cell consists of a resistor varying as a function of the applied force, and it is interfaced with a fixed resistor to form a voltage divider. The resulting voltage is acquired using the analog to digital conversion features of the acquisition card.

A simplified block diagram of the FSR interfacing system is given in Figure 1. The acquisition is realized sequentially: all the cells on a single line are read, then the line is changed. In order to successfully achieve this sensor acquisition method, a BCD decade counter (binary coded decimal, meaning this device counts from 0 to 9 with a binary encoding) is increased using a digital clock provided by the NI card. Then, an analog multiplexer is used in order to connect the cell to the fixed resistor to obtained the voltage divider.

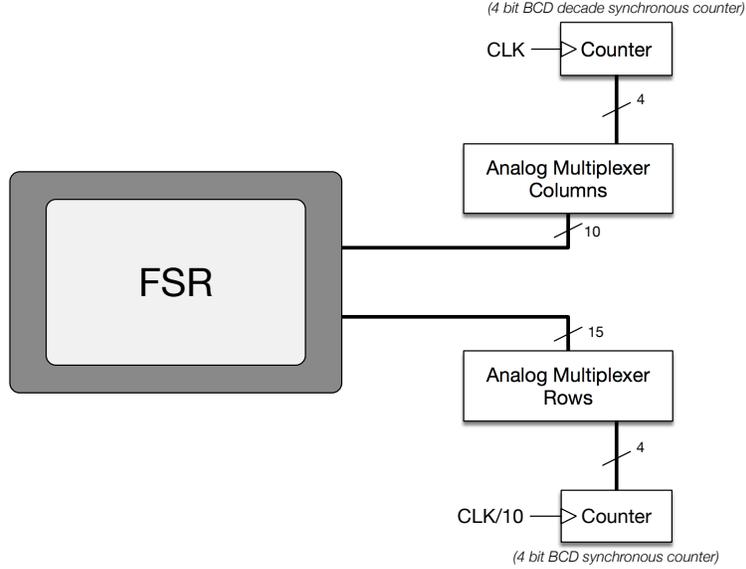


Figure 1: Block diagram of the grip force sensor

The method to change the lines is the same, except for the clock frequency, which is a tenth of the columns clock frequency. This frequency was set to 3,200 Hz for the columns, while it was of 320 Hz for the rows. As a result, for each cell, we acquire the following voltage:

$$V_{\text{ad}} = \frac{R_{\text{FSR}}}{R_{\text{FSR}} + R_{\text{ref}}} \cdot V_{\text{dd}} \quad (1)$$

where V_{ad} is the acquired voltage, R_{FSR} the resistance of the selected cell, which is a function of the force applied on the cell, R_{ref} is a 13 k Ω resistor and V_{dd} the supply voltage (5 V in our case).

In order to minimize the impact of settling time for each cell and to reduce the noise, we acquired three voltage samples for each cell, and then took the average. In order to make the preprocessing easier, a column was set to always be 0, because it allowed us to have a reference value each time all the columns were processed.

2.3. Sensor Calibration

Both the tablet and the FSR-based force sensor were then calibrated using reference weights. To calibrate the pen-tip normal force, we use a fixed vertical structure, and a slide link between the structure and the pen was created, allowing the pen to freely move along the tablet orthogonal axis. Reference weights were then placed on a known-weight vertical platform attached to the top of the pen, and the output of the tablet was then compared to the reference weight. A schematic of calibration apparatus is given in Figure 2. In order to calibrate the grip force sensor, an apparatus applying a known weight on a single cell was created, and was used to calibrate the sensor.

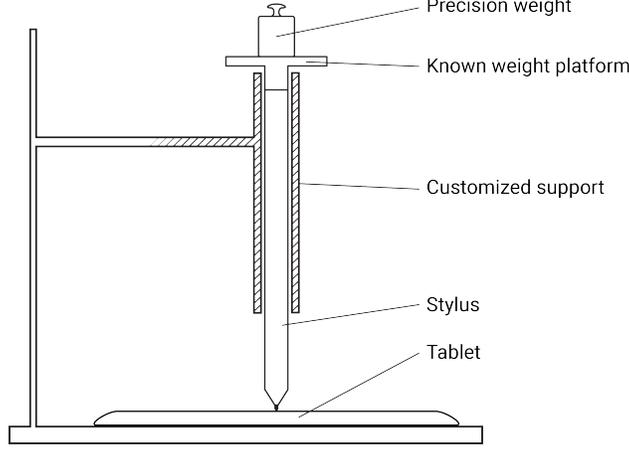


Figure 2: Calibration apparatus schematic

The expression resulting of the calibration of the tablet was obtained using a minimal squared error fitting of the calibration point we used and is given as:

$$F_{\text{tab}} = 43.18 \hat{n}^2 + 3.89 \hat{n} + 7 \quad (2)$$

where F_{tab} is the force in Newtons and $\hat{n} = n/1023$ is the normalized value returned by the tablet (n being the actual numerical value returned and 1023 the maximum value that the tablet can detect). For this calibration, the coefficient of determination was $R^2 = 0.98$.

The curve fitting procedure for the FSR-based sensor was different: because it was required to have a first order derivative of 0 at the origin in order to insure positive calibration results for small forces, we chose to set the calibration curve to be:

$$F_{\text{FSR}} = a \cdot (V_{\text{dd}} - V_{\text{ad}})^2 \quad (3)$$

The goal was to find the value of a maximizing the coefficient of variation R^2 . This coefficient is given by [16]:

$$R^2 = 1 - \frac{SS_E}{SS_{yy}} = 1 - \frac{\sum (y_i - \bar{y})^2}{\sum (y_i - \hat{y}_i)^2} \quad (4)$$

where y_i are the calibration force samples, \bar{y} is the average of the calibration force samples and \hat{y}_i is the value returned by the model, in our case $\hat{y}_i = (F_{\text{FSR}})_i = a \cdot (V_{\text{dd}} - (V_{\text{ad}})_i)^2$.

Then, the value of a maximizing the coefficient of variation was find using a simple iterative search. The optimal a was found as $a = 0.62$, with a coefficient of determination of $R^2 = 0.98$. To summarize, the calibration curve is:

$$F_{\text{FSR}} = 0.62 (V_{\text{dd}} - V_{\text{ad}})^2 \quad (5)$$

3. Feature Extraction

3.1. Pin-Tip Features

3.1.1. Normal Force Related Features

The mean, the coefficient of variation (the ratio of the standard deviation to the mean [27], denoted as CV), the skewness and the kurtosis of the normal force signal were extracted and analysed. The expression of a signal's $\mathbf{x} = [x_1, \dots, x_N]$ kurtosis is expressed as [17]:

$$\beta_2 = \frac{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^4}{\left[\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \right]^2} \quad (6)$$

where \bar{x} is the sample mean of the signal and N is its length. The expression of the signal's skewness is [17]:

$$\beta_1 = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^3}{\sqrt{\left[\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \right]^3}} \quad (7)$$

These parameters are an indicator of the signal distribution's shape [17]. The interquartile range (IQR), defined as the difference between the 75th and the 25th percentiles of the input data, was extracted.

3.1.2. Circle Characteristics

Several parameters reflecting both the speed and quality of the circles drawn by the subject were considered: radial error in the four quadrants positioned as displayed in 3, which was defined as the distance between the traced circle and the actual circle (in cm); the number of circles; the total path length in centimeters; and the average speed in cm.s^{-1} .

The radial error in the quadrants was computed using a linear translation along both axis, matching the origin of the axis to the center of the targeted circle, thus allowing a quadrant identification by a simple sign analysis on the coordinates of the measured point. The error was then computed for each quadrant using the following equation:

$$\epsilon_{i,n} = r_{\text{exp}} - r_{\text{th}} = \sqrt{x_{\text{exp}}^2 - y_{\text{exp}}^2} - r_{\text{th}} \quad (8)$$

where ϵ_i is the radial error of quadrant i at time instance n .

3.2. Grip Force Features

The first step of the feature extraction was to extract the total grip force signal from the FSR-based sensor. Consequently, the contribution of each of the cells of the FSR matrix at a given time were added [9]:

$$TGFS[t] = \sum_{i=1}^{16} \sum_{j=1}^9 (F_{\text{FSR}})_{i,j}[t] \quad (9)$$

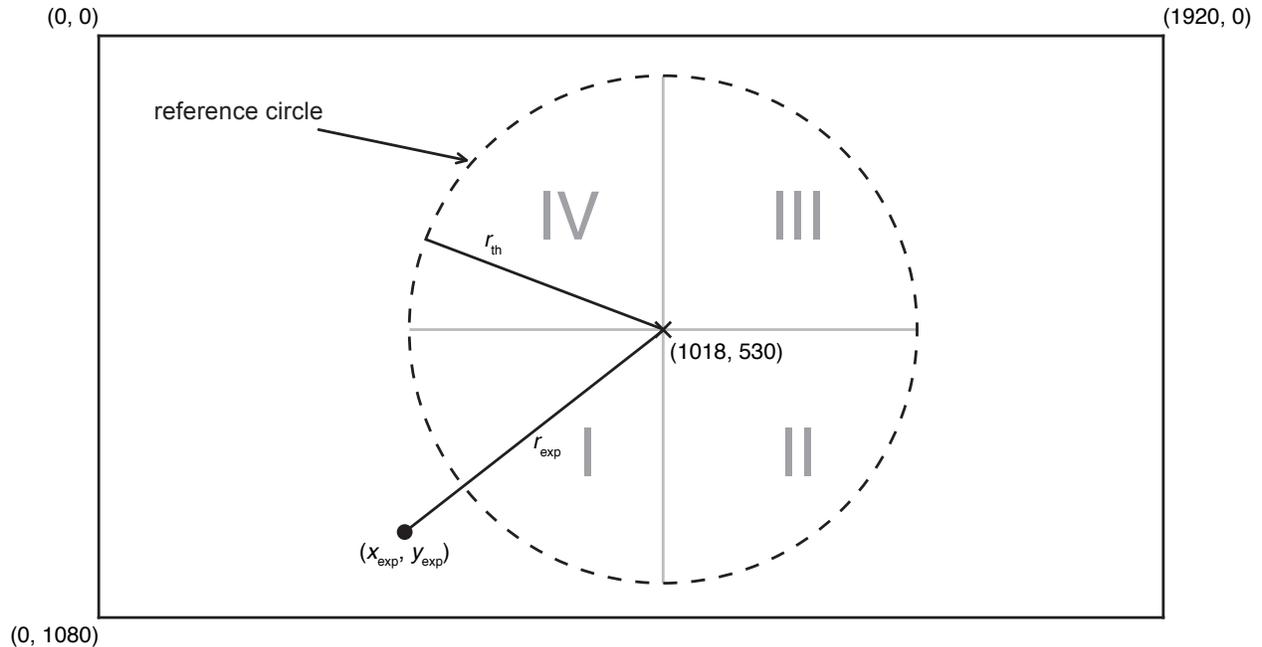


Figure 3: Illustration of the position of the quadrants

where i is the row address, j is the column address and $TGFS[t]$ the total grip force signal at discrete time t . Then, the mean, the coefficient of variation, the skewness, the kurtosis, and the interquartile range (IQR) of the total grip force signal were computed and statistically tested.

3.3. Statistical Analysis

To compare conditions, we fitted a series of linear mixed models with each measure of writing kinetics as the dependent variable, condition as the fixed effect of interest, and a participant random effect to account for multiple measurements from the same participant. We obtained type 3 p -values for the overall comparisons across the 4 conditions, and constructed means contrasts for pairwise comparisons. To examine dependence of writing kinetics on participant characteristics such as age, gender and BMI, we fitted a series of multiple regression models to each measure of writing kinetics with participant characteristics as independent variables and a stratification by condition. SAS[®] version 9.3 (SAS Institute, Inc., Cary, North Carolina) was used for all statistical analyses.

4. Results

The results are presented in figures where the tasks when the subject were not constrained by a metronome are referred as “Free” while the tasks where they were are referred as “Metro”. Figure 4 illustrates the collected signals.

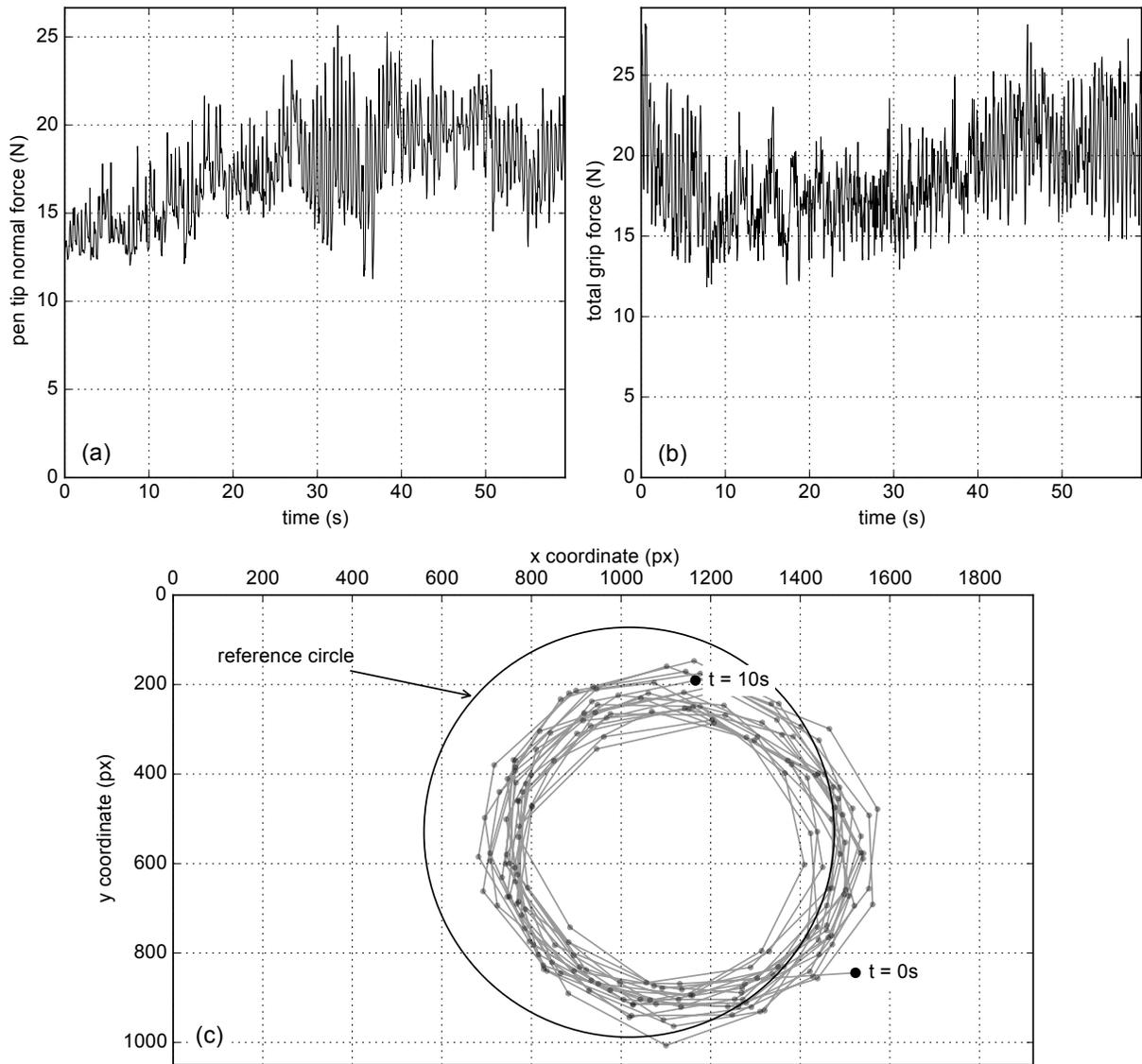


Figure 4: Examples of the collected signals after resampling, for subject 1, fast speed with metronome: pen tip normal force (a), total grip force (b) and drawn circles during the first 10 seconds (c)

4.1. Pen-Tip Normal Force

The results related to the pen-tip normal force are given in Figure 5. The mean pen-tip pressure was of about 15 N across all the 4 trials, showing important variability (i.e. an elevated standard deviation), meaning that this feature varies greatly between subjects. The relative variability of the signals, characterized by the CV [27], is in the of about 14%. This means that the pen grip force signal does not exhibit strong variations around its mean. For these features, nor pairwise or global statistical differences were found, meaning that nor the execution speed or the timing constraints influenced the pen-tip normal force in circle drawings.

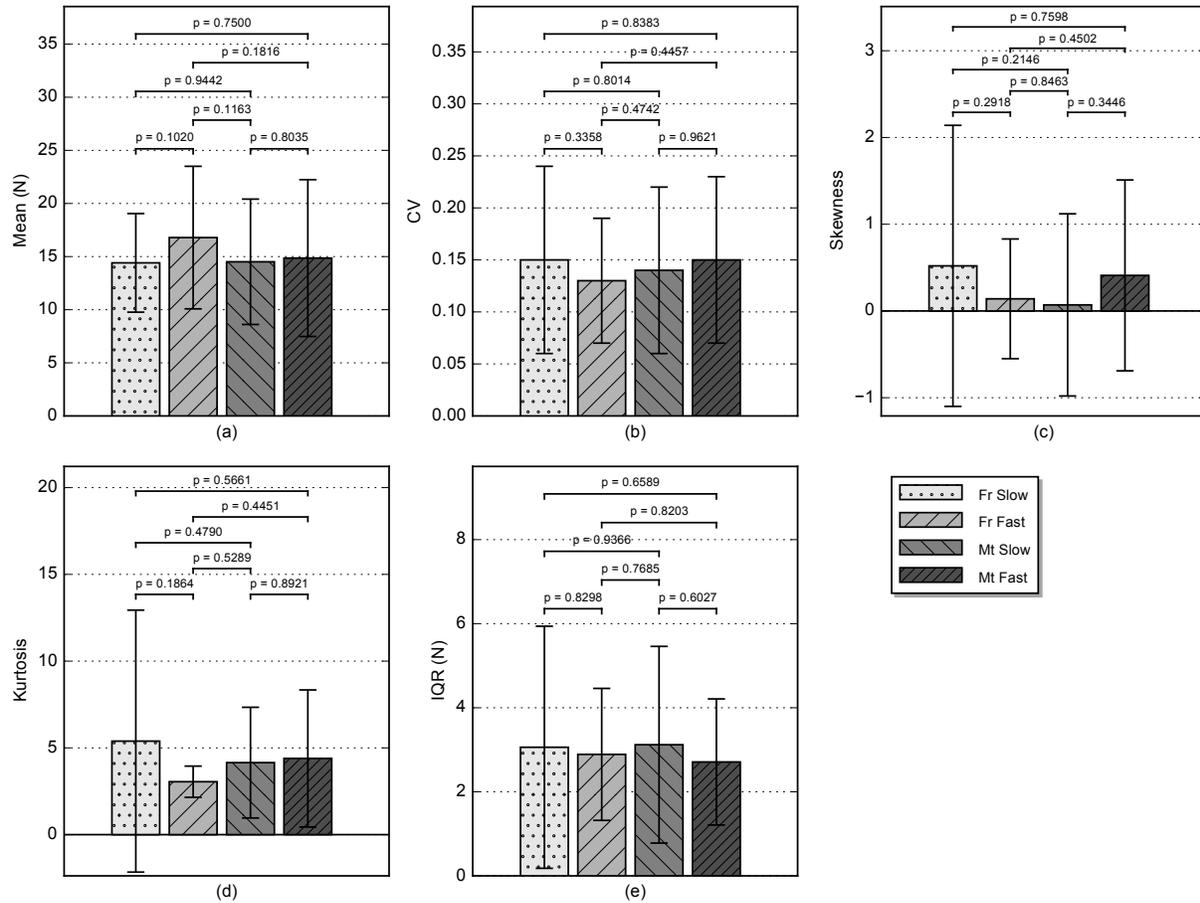


Figure 5: Mean, standard deviation and statistical analysis results for the tablet pressure features: force distribution mean (a), standard deviation (b), skewness (c), kurtosis (d) and inter-quartile range (e)

4.2. Total Grip Force

The results related to the total grip force signals are given in Figure 6. The mean, CV, kurtosis, skewness and IQR of the total grip force signal were found to behave like the pen-tip normal force signal features. We did not find any statistical differences for these features either meaning that for healthy subjects, nor the timing of the task or the speed impacts the total grip force signal.

4.3. Circle Results

The results related to the drawn circles are given in Figure 7. First, it can be observed that the speed changes (i.e. free fast vs free slow, free fast vs metro slow, free slow vs metro fast and metro slow vs metro fast, in other words statistical differences denoted as letter *a*, *c*, *d* and *f* in the table) caused statistical differences for the number of circles, the average speed and the total path length. This was expected, and it denotes that the subjects made a true distinction between a freely determined slow and fast speed. More

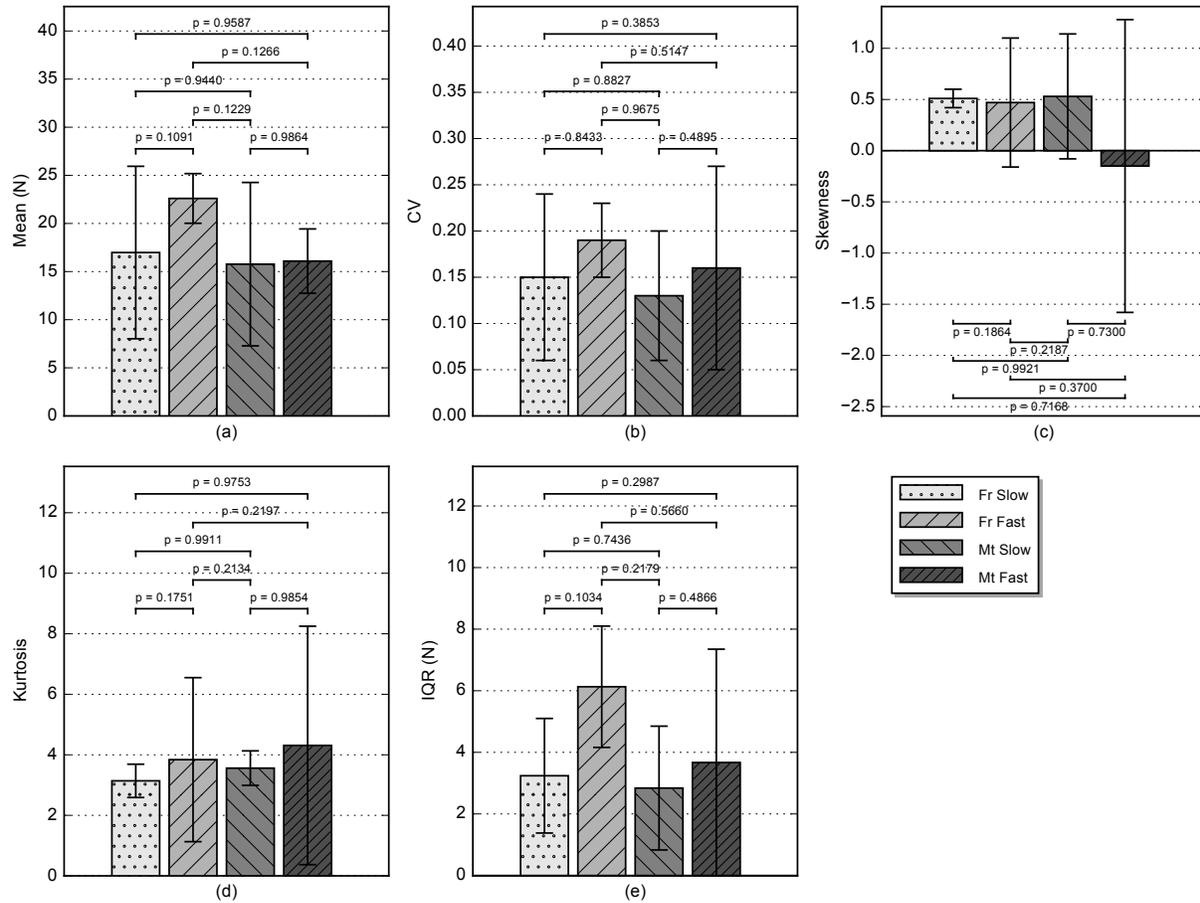


Figure 6: Mean, standard deviation and statistical analysis results for the total grip force features: total grip force distribution mean (a), standard deviation (b), skewness (c), kurtosis (d) and inter-quartile range (e)

interestingly, statistically significant changes were also in the radial errors (quadrants I, II and IV only). The quadrant II was only affected by speed differences (letters *a*, *c*, *d* and *f* in the table); while other quadrants were both influenced by some speed changes, but also by timing constraints changes. More specifically, statistical differences between both the freely determined slow and fast speed trials and respectively the time constrained slow and fast speed trials were found for quadrant I and IV (letters *b* and *e* in the table).

4.4. Simultaneous Comparison

The results of the simultaneous comparison across the four trials are given in Table 1. Since no statistical differences were found for the pen-tip normal force and total grip force features, they are not displayed in this table. These global statistical differences matched the pairwise statistical differences illustrated in Section 4.3.

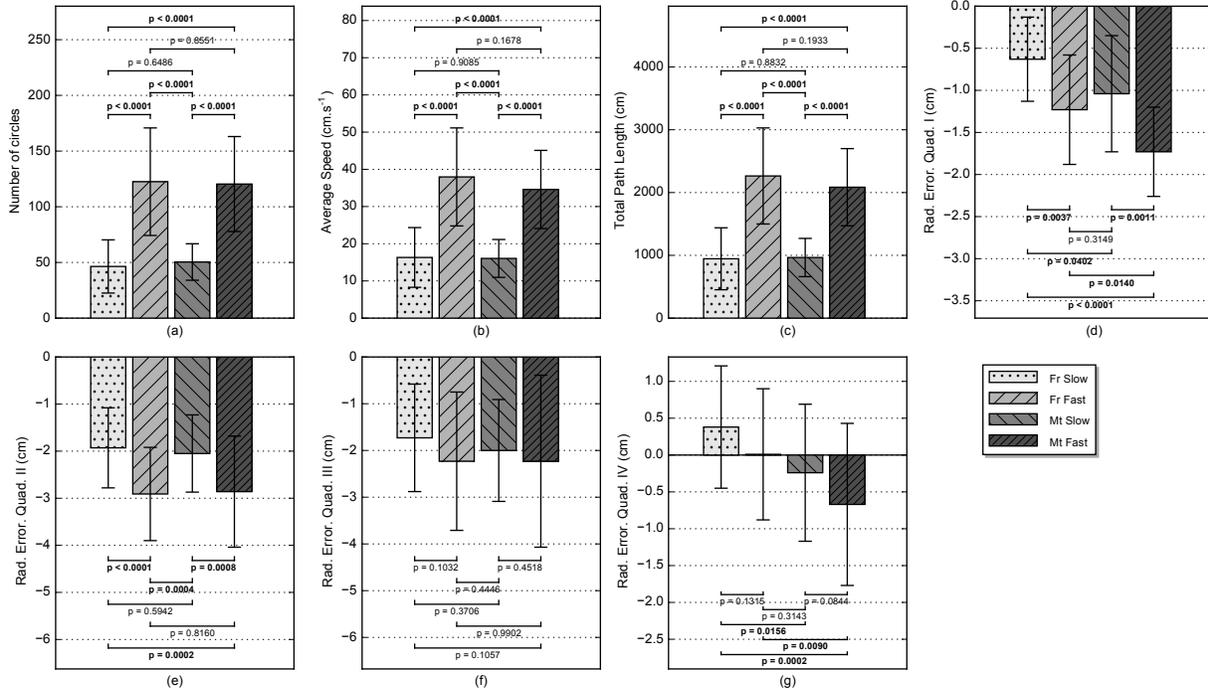


Figure 7: Mean, standard deviation and statistical analysis results for the circle measures features: number of circles (a), average speed (b), total path length (c) and radial error in quadrants I to IV (d – g)

4.5. Impact of the Anthropometric Variables

The impact of the anthropometric variables is given in Table 2. Statistically significant impact of these variable were only found on the pen-tip normal force skewness in both the free trials, and only for the skewness in the free slow trial.

The anthropometric variables were not found to have an impact on the total grip force signal.

5. Discussion

The global statistically significant increase of the radial error in quadrant I and II cause by an increase in the speed suggest that even though nor the grip force or the pen-tip normal force are impacted by the velocity change, a decrease of the precision of the movement can be observed as speed increases. This speed-accuracy trade-off have already been found for manual prehension movements [3], but also for more general target directed movements [20] or rotary movements of the wrist [14], and we found that these trade-offs are also verified in the writing kinetics case.

The cerebellum seems to also have a role in the control of this speed-accuracy trade-off [23, 4]. This trade-off for targeted movements can be summarized by Fitts' law [8, 4]: this law establishes an affine relationship between the movement time and the logarithm of the ratio of the distance of the aimed target

Table 1: Simultaneous comparison between the four conditions

Global Statistical Difference	
Number of Circles	★ ($p < 0.0001$)
Average Speed	★ ($p < 0.0001$)
Total Path Length	★ ($p < 0.0001$)
Rad. Err. Quad. I	★ ($p < 0.0001$)
Rad. Err. Quad. II	★ ($p < 0.0001$)
Rad. Err. Quad. III	
Rad. Err. Quad. IV	★ ($p = 0.015$)

Table 2: Impact of the antropometric variables on the features

	Free Slow	Free Fast	Metro Slow	Metro Fast
Pen Features				
Mean				
CV		★ ($p = 0.0466$)		
Skewness	◆ ($p = 0.0489$)	★ ⊗ ◆ ($p < 0.0004$)		
Kurtosis				
IQR				
TGF Features				
Mean				
CV				
Skewness				
Kurtosis				
IQR				
Circle Features				
Number of Circles				
Average Speed				★ ($p = 0.0439$)
Total Path Length				
Rad. Err. Quad. I				
Rad. Err. Quad. II				
Rad. Err. Quad. III				
Rad. Err. Quad. IV				

★ denotes the statistical influence of gender

⊗ denotes the statistical influence of age

◆ denotes the statistical influence of BMI

and its size. In our case, the distance to the next target is the distance to the next targeted point of the circle and the width of this target is the pen-tip size, that has to match the circle's line arbitrarily small size.

If we consider a decrease of the movement time (i.e. a faster drawing speed), and since the distance to the target is fixed, the target’s size must increase, thus reducing the accuracy.

Moreover, the imposed timing of the task also provided statistical differences for some quadrants, pointing toward a “timing-accuracy” trade-off, where the constrained timing of a task impacts the accuracy of its realization. This timing-accuracy trade-off might originate in the timing nature differences between the two tasks. Indeed, the task performed without the assistance of the metronome can be described as an implicit timed task, while the task where the metronome is used and where the time per circle is constrained can be qualified as an explicit timed task. Implicit timing is defined as tasks where the explicit representation of time does not influence the performance of the task, while explicit timing refers to tasks where the performance is strongly linked to the explicit representation of time [28].

The fundamental difference between these two tasks resides in the biological origin of the timing control. Indeed, it was shown that central timing (i.e. explicit representation of time) and the implementation of timing process (i.e. the implicit timing) are independent, and originates in different brain regions: the explicit representation of the time (i.e. central timing) was shown to result of the lateral region of the cerebellum, while the implementation of a time sensitive task originates from the medial region [11].

Both this difference of physiological origin or more high-level timing perception origin could explain the differences between the time constraints free tasks and the time constrained task in terms of accuracy.

It is also of interest to notice that quadrant III is not displaying any statistically significant of nor timing or speed: this quadrant match the phase when the subjects realizes a compression motion, where the wrist and fingers are the closest to the elbow. This points towards an influence of the nature of the movement on its accuracy: extension (i.e. reaching) movements seemed to be impacted by speed and timing, while compression movements were not. This is in agreement were the studies showing a speed-accuracy trade-off for prehension movements, where the movement is clearly of expensive nature.

The main limitation of this paper is the relatively small cohort, but it needs to be pointed out that our primary goal was to explore the creation of a new writing analysis tool, and the second goal was to report any significant finding in the data collected using our newly designed apparatus. The smaller cohort is justified as it would have been ill-suited to collect data with an unverified apparatus. Moreover, we believe the advanced statistical analysis performed in our paper provides relevant insights into the significance of the various hypothesis despite the relatively small cohort.

6. Conclusion

In this paper, we simultaneously studied pen-tip normal force characteristics, total grip force features and the quality of the resulting writing during four circle drawing tasks. These tasks were designed to investigate both the impact of speed and timing constraints on the writing characteristics described above. We found

that while nor speed or timing constraints changes impacted pen-tip normal force and total grip force signal, they did impact writing quality features, more particularly they on average increased error between the traced circles and the intended circles. This confirms the existence of a speed-accuracy trade-off in writing, but also suggests the existence of a timing constraints-accuracy trade-off.

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