Instantaneous Frequency Estimation Using the S-Transform

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***Abstract*—Instantaneous frequency (IF) is a fundamental concept that can be found in many disciplines such as communications, speech, and music processing. In this letter, analysis of an IF estimator, based on a time-frequency technique known as S-transform, is performed. The performance analysis is carried out in a white Gaussian noise environment, and expressions for the bias and the variance of the estimator are determined. The results show that the bias and the variance are signal dependent. This has been statistically confirmed through numerical simulations of several signal classes.**

***Index Terms*—Instantaneous frequency estimation S-transform, time-frequency.**

I. INTRODUCTION

INSTANTANEOUS frequency (IF), defined as the derivative of the phase of a signal, is a fundamental concept present not only in communications (e.g., frequency modulation) but also in nature (e.g., changing color of light) [1]. The estimation of IF is important in signal analysis with time-frequency analysis being one of the tools for IF estimation [2].

The S-transform is conceptually a hybrid of short-time Fourier analysis and wavelet analysis. It employs a variable window length but preserves the phase information by using the Fourier kernel in the signal decomposition [3]. As a result, the phase spectrum is absolute in the sense that it is always referred to a fixed time reference. The real and imaginary spectrum can be localized independently with resolution in time in terms of basis functions. The changes in the absolute phase of a certain frequency can be tracked along the time axis and useful information can be extracted. For this reason, the S-transform has already found applications in many fields such as geophysics [4], cardiovascular time series analysis [5], pattern recognition [6], and signal processing for mechanical systems [7].

The IF of a noiseless signal can be determined from the phase of the signal representation obtained by the S-transform. It is given by a partial derivative of the phase with respect to time [3]. However, in previous works, it has not been demonstrated how well the S-transform performs when a signal is contaminated by noise. Hence, the main contribution of this letter is the derivation of accurate analytical expressions for instantaneous frequency estimation error using the S-transform. Such an analysis is performed for signals in the presence of white Gaussian noise. Results of numerical analysis confirm the derived expressions.

This letter is organized as follows: In Section II, a review of the S-transform followed by derivations of the bias and the variance for the IF estimation algorithm are given. Section III illustrates the performance of the proposed scheme through an example. Finally, conclusions are drawn in Section IV.

II. PERFORMANCE ANALYSIS OF THE S-TRANSFORM-BASED IF ESTIMATOR

A. S-*Transform*

The S-transform of a signal $x(t)$ is defined by [3]

$$S_c(t,\omega;w(\tau,\omega)) = e^{-j\omega t}\int_{-\infty}^{+\infty} x(t+\tau)w(\tau,\omega)e^{-j\omega\tau}d\tau \quad (1)$$

where the window function is

$$w(\tau,\omega) = \frac{|\omega|}{(2\pi)^{1.5}}e^{-\frac{\tau^2\omega^2}{8\pi^2}}. \quad (2)$$

If the discrete samples of the continuous signal are available, the integral form of the S-transform can be discretized. By sampling (1) in τ with a sampling period T, a discretized S-transform can be defined as follows:[1]

$$S_d(t,\omega;w(nT,\omega)) = e^{-j\omega t}T\sum_n x(t+nT)w(nT,\omega)e^{-j\omega nT}. \quad (3)$$

Due to the fact that the window is a function of both time and frequency, it can be noted that the window is wider in the time domain for lower frequencies, and it is narrower for higher frequencies. In other words, a wide time-domain window implies good localization in the frequency domain for low frequencies, while a narrow window provides good localization in the time domain for higher frequencies.

In order to illustrate the advantage of the variable window length used in the S-transform, let us compare the time-frequency representations obtained by the short-time Fourier transform (STFT), pseudo Wigner–Ville distribution (PWVD), and the S-transform for the following signal:

$$x(t) = \begin{cases} \exp(-j10\pi\ln(-25t+1)) & -2 \le t \le 0 \\ \exp(j10\pi\ln(25t+1)) & 0 < t < 2 \end{cases} \quad (4)$$

where the IF of the signal is given by $\omega(t) = 250\pi/(25|t|+1)$ for $-2 \le t < 2$. This is only an illustrative example; more thorough investigation of hyperbolic FM signals and other signal classes will be presented in Section III.

[1] $\sum_n \equiv \sum_{n=-\infty}^{+\infty}$ unless otherwise stated.

Manuscript received August 29, 2007; revised November 7, 2007. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. James E. Fowler.

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Digital Object Identifier 10.1109/LSP.2008.917014

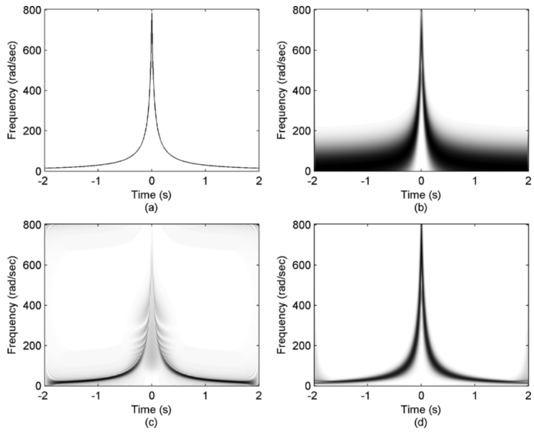

Fig. 1. Four time-frequency representations of a sample hyperbolic signal. (a) Ideal time-frequency representation. (b) STFT. (c) PWVD. (d) S-transform.

TABLE I
MEAN-SQUARE ERROR FOR THE IF ESTIMATION BASED ON THE THREE TIME-FREQUENCY REPRESENTATIONS UNDER THREE NOISE LEVELS

TFR	Noise $\sigma^2 = 0$	Noise $\sigma^2 = 0.1$	Noise $\sigma^2 = 0.2$
STFT	7.0975	7.1106	7.3632
PWVD	0.9108	0.9365	1.1525
S-transform	0.1120	0.1253	0.1403

The Gaussian window is also used for the STFT and the PWVD, with the standard deviations of the window being 0.01 and 0.25, respectively. The sampling interval is $T = 1/256$ se. The advantage of the S-transform as the instantaneous frequency estimator can be seen from Fig. 1. In order to obtain good concentration at higher frequencies, a narrow window in the time domain should be used for the STFT. However, the narrow window in the time domain significantly diminishes the concentration of the lower frequencies contents of the signal as shown in Fig. 1(b). Even though the PWVD provides improvement over the Wigner–Ville distribution and achieves higher concentration for all frequencies than the STFT, it still suffers from some inner interference effects and cross-terms. The S-transform provides much better representation of the sample hyperbolic signal as shown in Fig. 1(d).

It is also important to examine the mean-square error (MSE) of the IF estimator based on the STFT, the PWVD, and the S-transform. The MSE is defined as $E\left\{(\omega(t) - \hat{\omega}(t))^2\right\}$, where $\omega(t)$ is the IF of the signal and $\hat{\omega}(t)$ is its estimate. The IF is estimated based on the peak of the magnitude of each transform with the additive complex white Gaussian noise with the total variance of σ^2 added to the signal. The MSE in Table I represent an average of 1000 realizations. These results demonstrate that the MSE of the IF estimator based on the S-transform is significantly lower than those based on STFT and PWVD.

B. Performance Analysis

In order to perform a statistical analysis of the estimator, discrete-time observations

$$x(nT) = f(nT) + \epsilon(nT) \tag{5}$$

will be used, where n is an integer, $f(nT)$ is a sampled version of a continuous analytical signal $f(t) = Ae^{j\phi(t)}$ with T being a sampling interval, and $\epsilon(nT)$ is a complex-valued white Gaussian noise with independent and identically distributed real and imaginary parts. Thus, $\Re(\epsilon(nT))$ and $\Im(\epsilon(nT)) \sim N(0, \sigma_\epsilon^2/2)$ and the total variance of the noise is equal to σ_ϵ^2.

By definition, the instantaneous frequency of the considered continuous signal is $\omega(t) = \mathrm{d}\phi(t)/\mathrm{d}t$, and it is assumed that $\omega(t)$ is an arbitrary smooth differentiable function of time with bounded derivatives $\left|\omega^{(r)}(t)\right| < \infty$, where $r \geq 1$ and $\omega^{(r)}(t)$ denotes an rth derivative of $\omega(t)$. The value of $\omega(t)$ can be estimated in the time-frequency domain as in the following [2]:

$$\hat{\omega}(t) = \arg\left[\max_{\omega \in Q_\omega} \Lambda(t, \omega)\right] \tag{6}$$

with $Q_\omega = \{\omega : 0 \leq \omega \leq \pi/T\}$ being a basic interval along the frequency axis, and where we see (7) at the bottom of the page.

Before proceeding further, let us consider $\Lambda(t, \omega)$ for some signal $f(t)$. Using the fact that the signal has a slow-varying amplitude and Taylor series expansion of the phase differences $f(t + n_1T)f^*(t + n_2T) = A^2 e^{j\phi(t+n_1T) - j\phi(t+n_2T)}$, $\Lambda(t, \omega)$ of $f(t)$ can be expressed as (8) at the bottom of the page, where $\Phi(n_1T, n_2T, t)$ equals the Taylor series expansion of the phase difference evaluated for the first M desired terms, that is, $k = 0, 1, \ldots, M$

$$\Phi(n_1T, n_2T, t) = \sum_{k=0}^{M} \phi^{(k)}(t) \frac{(n_1T)^k - (n_2T)^k}{k!} \tag{9}$$

and $\Delta\phi(n_1T, n_2T, t)$ equals the Taylor series expansion of the phase difference evaluated for $k = M + 1, M + 2, \ldots, \infty$

$$\Delta\phi(n_1T, n_2T, t) = \sum_{k=M+1}^{\infty} \phi^{(k)}(t) \frac{(n_1T)^k - (n_2T)^k}{k!} \tag{10}$$

with $\phi^{(k)}(t)$ representing a kth derivative of $\phi(t)$. In this letter, $\Delta\phi(n_1T, n_2T, t)$ represents the third and higher order terms, i.e., $M = 2$.

As a measure of the quality of estimation, at a given instant t, the estimation error can be defined as

$$\triangle\hat{\omega}(t) = \omega(t) - \hat{\omega}(t) \tag{11}$$

$$\Lambda(t, \omega) = S_d S_d^* = T^2 \sum_{n_1} \sum_{n_2} x(t + n_1T) x^*(t + n_2T) \times w(n_1T, \omega) w^*(n_2T, \omega) e^{-j\omega(n_1 - n_2)T}. \tag{7}$$

$$\Lambda(t, \omega) = T^2 \sum_{n_1} \sum_{n_2} A^2 e^{j\Phi(n_1T, n_2T, t) + j\Delta\phi(n_1T, n_2T, t)} \times w(n_1T, \omega) w^*(n_2T, \omega) e^{-j\omega(n_1 - n_2)T} \tag{8}$$

where $\omega(t)$ is the IF of the signal and $\hat{\omega}(t)$ is its estimate from the noisy observations of the signal. Due to the presence of the white Gaussian noise, the estimation error, $\triangle\hat{\omega}(t)$, is also a random variable characterized by its bias and variance.

Proposition: Let $\hat{\omega}(t)$ be a solution of (6); then the bias and the variance of the IF estimation error $\triangle\hat{\omega}(t)$ are given by

$$\text{bias}\ (\triangle\hat{\omega}(t)) = \frac{2\,\text{Re}\{P(t,\omega)E(t,\omega)\}}{4\pi^2T^{-1}\omega^{-2}E(t,\omega)-|P(t,\omega)|^2} + \frac{\text{Re}\{Q(t,\omega)F(t,\omega)\}+(8T)^{-1}A^{-2}\pi^{-3/2}\sigma^2}{4\pi^2T^{-1}\omega^{-2}E(t,\omega)-|P(t,\omega)|^2} \quad (12)$$

and

$$\begin{aligned}\text{var}\,(\triangle\hat{\omega}(t)) &= 2\sigma^2\,\text{Re}\{P(t,\omega)E(t,\omega)\} \\ &\times\left\{16\pi^{3/2}TA^2\left[|P(t,\omega)|^2-4\pi^2T^{-1}\omega^{-2}E(t,\omega)\right]^2\right\}^{-1} \\ &+\frac{\sigma^2\left(2\omega|P(t,\omega)|^2+\left(1.5+4\pi^2\right)\omega^{-1}|E(t,\omega)|^2\right)}{16\pi^{3/2}TA^2\left[|P(t,\omega)|^2-4\pi^2T^{-1}\omega^{-2}E(t,\omega)\right]^2} \\ &+\frac{\sigma^4\left(1+\pi^2\right)}{16\pi^3T^2A^4\left[|P(t,\omega)|^2-4\pi^2T^{-1}\omega^{-2}E(t,\omega)\right]^2}\end{aligned} \quad (13)$$

where

$$E(t,\omega)=\sum_n e^{-j\phi^{(2)}(t)((nT)^2/2)}w(nT,\omega) \quad (14)$$

$$F(t,\omega)=\sum_n e^{-j\phi^{(2)}(t)((nT)^2/2)-j\Delta\phi(nT,t)}w(nT,\omega) \quad (15)$$

$$P(t,\omega)=\sum_n e^{j\phi^{(2)}(t)((nT)^2/2)}\frac{\partial w(nT,\omega)}{\partial\omega} - \sum_n e^{j\phi^{(2)}(t)((nT)^2/2)}jnTw(nT,\omega) \quad (16)$$

$$Q(t,\omega)=\sum_n e^{j\phi^{(2)}(t)((nT)^2/2)+j\Delta\phi(nT,t)}\frac{\partial w(nT,\omega)}{\partial\omega} + \sum_n e^{j\phi^{(2)}(t)((nT)^2/2)+j\Delta\phi(nT,t)}\left(-jnTw(nT,\omega)\right) \quad (17)$$

with $\omega=\phi^{(1)}(t)$.

Proof: The IF is located at the stationary points of $\Lambda(t,\omega)$, which is defined by the zero value of the derivative $\partial\Lambda(t,\omega)/\partial\omega$, given by

$$\begin{aligned}\frac{\partial\Lambda(t,\omega)}{\partial\omega} &= jT^2\sum_{n_1}\sum_{n_2}x(t+n_1T)x(t+n_2T)(n_2-n_1) \\ &\times Tw(n_1T,\omega)w^*(n_2T,\omega)e^{-j\omega(n_1-n_2)T} \\ &+T^2\sum_{n_1}\sum_{n_2}x(t+n_1T)x(t+n_2T)\frac{\partial w(n_1T,\omega)}{\partial\omega} \\ &\times w^*(n_2T,\omega)e^{-j\omega(n_1-n_2)T} \\ &+T^2\sum_{n_1}\sum_{n_2}x(t+n_1T)x(t+n_2T)w(n_1T,\omega) \\ &\times\frac{\partial w^*(n_2T,\omega)}{\partial\omega}e^{-j\omega(n_1-n_2)T}.\end{aligned} \quad (18)$$

To perform the estimation error analysis, $\partial\Lambda(t,\omega)/\partial\omega$ is linearized around the stationary point with respect to small estimation error $\triangle\hat{\omega}(t)$, phase residue $\triangle\phi$, and noise ϵ [2]

$$\left.\frac{\partial\Lambda(t,\omega)}{\partial\omega}\right|_0+\left.\frac{\partial^2\Lambda(t,\omega)}{\partial\omega^2}\right|_0\triangle\hat{\omega}(t) + \left.\frac{\partial\Lambda(t,\omega)}{\partial\omega}\right|_{0\delta_{\triangle\phi}}+\left.\frac{\partial\Lambda(t,\omega)}{\partial\omega}\right|_{0\delta_\varepsilon}=0 \quad (19)$$

where $|_0$ indicates that the derivatives are evaluated at the point $\omega=\phi'(t)$, $\triangle\phi=0$, and $\epsilon=0$. The terms $\partial\Lambda(t,\omega)/\partial\omega|_{0\delta_{\triangle\phi}}$ and $\partial\Lambda(t,\omega)/\partial\omega|_{0\delta_\epsilon}$ represent variations of the derivative $\partial\Lambda(t,\omega)/\partial\omega$ caused by small $\Delta\phi(n_1T,n_2T,t)$ and noise $\epsilon(nT)$, respectively. It can be shown that the terms in (19) are equal to

$$\left.\frac{\partial\Lambda(t,\omega)}{\partial\omega}\right|_0=2T^2A^2\Re\{P(t,\omega)E(t,\omega)\} \quad (20)$$

$$\left.\frac{\partial^2\Lambda(t,\omega)}{\partial\omega^2}\right|_0=\frac{2TA^2}{\omega^2}\left[T\omega^2|P(t,\omega)|^2-4\pi^2E(t,\omega)\right]. \quad (21)$$

Similarly, the effects of a phase residue, that is, third and higher order terms in a Taylor expansion of the phase $\phi(t)$, are given by

$$\left.\frac{\partial\Lambda(t,\omega)}{\partial\omega}\right|_{0\delta_{\triangle\phi}}=2T^2A^2\Re\{Q(t,\omega)F(t,\omega)\}. \quad (22)$$

The expected value of the last term in (19) is given by

$$E\left\{\left.\frac{\partial\Lambda(t,\omega)}{\partial\omega}\right|_{0\delta_\varepsilon}\right\}=\left.\frac{\partial\Lambda(t,\omega)}{\partial\omega}\right|_0+\frac{\sigma^2T}{4\pi^{3/2}} \quad (23)$$

and the expected value of its square is given by

$$\begin{aligned}E\left\{\left(\left.\frac{\partial\Lambda(t,\omega)}{\partial\omega}\right|_{0\delta_\epsilon}\right)^2\right\} &= \left(\left.\frac{\partial\Lambda(t,\omega)}{\partial\omega}\right|_0\right)^2+\frac{3T^2\sigma^4}{32\pi^3} \\ &+\frac{T^3\sigma^2A^2\omega}{2\pi^{3/2}}|P(t,\omega)|^2+\frac{2T^3\sigma^2\left(3+8\pi^2\right)A^2}{8\pi^{3/2}\omega}|E(t,\omega)|^2 \\ &+\frac{12T^3A^2\sigma^2}{8\pi^{3/2}}\text{Re}\{P(t,\omega)E(t,\omega)\}+\frac{T^2\sigma^4\left(3+8\pi^2\right)}{32\pi^3}.\end{aligned} \quad (24)$$

Having obtained the expressions for all the terms in (19), it can be written as

$$\triangle\hat{\omega}(t)=\frac{-\left.\frac{\partial\Lambda(t,\omega)}{\partial\omega}\right|_0-\left.\frac{\partial\Lambda(t,\omega)}{\partial\omega}\right|_{0\delta_{\triangle\phi}}-\left.\frac{\partial\Lambda(t,\omega)}{\partial\omega}\right|_{0\delta_\varepsilon}}{\left.\frac{\partial^2\Lambda(t,\omega)}{\partial\omega^2}\right|_0}. \quad (25)$$

By substituting (20)–(23) in (25), an expression for the bias of the estimation is obtained as

$$\text{bias}\ (\triangle\hat{\omega}(t)) = \frac{2\,\text{Re}\{P(t,\omega)E(t,\omega)\}}{4\pi^2T^{-1}\omega^{-2}E(t,\omega)-|P(t,\omega)|^2} + \frac{\text{Re}\{Q(t,\omega)F(t,\omega)\}+(8T)^{-1}A^{-2}\pi^{-3/2}\sigma^2}{4\pi^2T^{-1}\omega^{-2}E(t,\omega)-|P(t,\omega)|^2} \quad (26)$$

which is equal to (12). To determine the variance of the estimation, the following relation is used:

$$\text{var}\,(\triangle\hat{\omega}(t))=E\left\{[\triangle\hat{\omega}(t)]^2\right\}-E^2\{\triangle\hat{\omega}(t)\}. \quad (27)$$

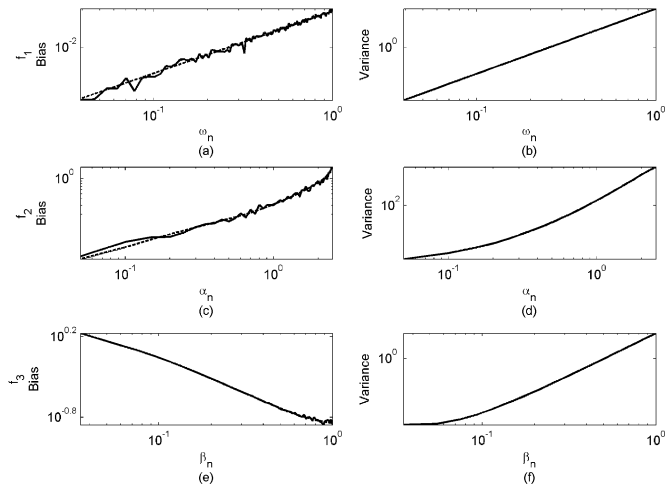

Fig. 2. Performance of IF estimator using the S-transform. (a) Bias of the estimator for CS signal. (b) Variance of the estimator for CS signal. (c) Bias of the estimator for the LFM signal. (d) Variance of the estimator for the LFM signal. (e) Bias of the estimator for the HFM signal. (f) Variance of the estimator for the HFM signal.

Equations (25) and (26) are substituted in (27), and after some algebraic manipulations, the following expression is obtained:

$$\begin{aligned}
&\operatorname{var}(\triangle\hat{\omega}(t)) \\
&= \frac{E\left\{\left(\left.\frac{\partial\Lambda(t,\omega)}{\partial\omega}\right|_{0\delta_\varepsilon}\right)^2\right\} - E\left\{\left.\frac{\partial\Lambda(t,\omega)}{\partial\omega}\right|_{0\delta_\varepsilon}\right\}^2}{\left(\left.\frac{\partial^2\Lambda(t,\omega)}{\partial\omega^2}\right|_0\right)^2} \\
&= \frac{2\sigma^2\,\mathrm{Re}\{P(t,\omega)E(t,\omega)\}}{16\pi^{3/2}TA^2\left[|P(t,\omega)|^2 - 4\pi^2T^{-1}\omega^{-2}E(t,\omega)\right]^2} \\
&\quad + \frac{\sigma^2\left(2\omega|P(t,\omega)|^2 + \left(1.5+4\pi^2\right)\omega^{-1}|E(t,\omega)|^2\right)}{16\pi^{3/2}TA^2\left[|P(t,\omega)|^2 - 4\pi^2T^{-1}\omega^{-2}E(t,\omega)\right]^2} \\
&\quad + \frac{\sigma^4\left(1+\pi^2\right)}{16\pi^3T^2A^4\left[|P(t,\omega)|^2 - 4\pi^2T^{-1}\omega^{-2}E(t,\omega)\right]^2}. \quad (28)
\end{aligned}$$

It is clear that (28) is equal to (13). ■

III. Example

In the analysis, three classes of signals are used. A simple complex sinusoidal (CS) signal $f_1(t) = \exp(j\omega t)$, a linear FM (LFM) signal $f_2(t) = \exp\left(j80\pi t + j(\alpha/2)t^2\right)$, and a hyperbolic FM (HFM) signal $f_3(t) = \exp(j\beta\ln(t+1))$ are considered. The sampling interval used is $T = 1/256$ with 2048 data points, and the variance of the noise used in the analysis is set to $\sigma_\epsilon^2 = 0.1$. The results of the numerical analysis along with the theoretical values are depicted in Fig. 2, and they represent the bias and the variance for $S(1,\omega)$. Values from the analytical expressions are produced by applying the derived expressions (12) and (13) for the particular signals, while the statistical data are obtained by 100 000 realizations. The vertical axis represents the magnitude of the estimation bias and variance, while the horizontal axis represents the values of variables (ω, α, and β) normalized with the Nyquist frequency for the sampling interval T ($\omega_n = \omega T/\pi$, $\alpha_n = \alpha T/\pi$, and $\beta_n = \beta T/\pi$).

A good agreement between the theoretical results (dashed line) and the statistical results (solid line) can be observed. For all three classes of the signals, as the frequency of the signals increases, the variance of the of the IF estimation error increases, too. For the complex sinusoids, the variance exhibits linear behavior, while for the rest, the variance behaves in a nonlinear manner. The bias linearly increases for the complex sinusoids and the LFM signals. However, as β increases, the bias linearly decreases for the HFM signals due to the fact that the S-transform achieves higher energy concentration of the signal for the given sampling interval. Hence, the peak location, that is, the instantaneous frequency, is estimated more accurately. This essentially means that a linear time-frequency representation such as the S-transform could be potentially used for accurate estimation of the instantaneous frequency for certain higher order signals which previously has been only accomplished with bilinear representations.

IV. Conclusion

In this letter, the analysis of an IF estimation based on the S-transform has been performed. Such an analysis has been carried out for signals in the presence of white Gaussian noise. Accurate analytical expressions for instantaneous frequency estimation error using the S-transform have been derived. These expressions have been confirmed through numerical analysis. Furthermore, the analysis has shown that the bias and the variance are signal dependent. For the considered signal classes, the S-transform showed favorable performance for the HFM signals.

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